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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 4: combining terms, equations with variables in both members, directed numbers, comparing numbers, addition on the number line, opposites, and absolute value. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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VOLUME

4

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

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PROGRAMMED MATH CONTINUUM

LEVEL ONE

A L G E B R A

VOLUME 4

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME IV

TABLE OF CONTENTS

	PAGE
COVER	
PREFACE	A
TABLE OF CONTENTS	B
SYLLABUS	C
READING ASSIGNMENT	D
HOMEWORK ASSIGNMENT	E
GENERAL INSTRUCTIONS	F

IN THE STUDY GUIDE:

QUESTION:	SEGMENT:	IS ON PAGE:
1	1	$\frac{1}{1}$
1	2	$\frac{40}{1}$
1	3	$\frac{80}{1}$
1	4	$\frac{134}{1}$
1	5	$\frac{171}{1}$

VOLUME 4

This volume covers the following material as shown in this excerpt from the Syllabus:

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION		
		DOLCIANI	DRESSLER	DODES
1	Combining terms, transformation	3-7	5-3 5-7 to 5-9 9-1 to 9-3	5-3
2	Equations-variable in both members	3-8	9-4	5-3
3	Directed members	4-1	6-1 6-2	3-3
	Comparing Numbers	4-2	6-3	3-3
4	Addition on number line	4-3	6-6	3-4
5	Opposite of directed number	4-4	6-4	3-4
	Absolute value	4-5	6-5	3-7

READING ASSIGNMENT

VOLUME IV

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	86	88
2	91	93
3	111	115
4	116	119
5	120	123

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1957

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

HOMEWORK ASSIGNMENT

VOLUME NO. 4

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	88	1 - 5	04120
2	88	6 - 10	04120
3	88	29 - 33	04120
4	89	5 - 11	04130
5	93 bottom	1 - 5	04210
6	93 bottom	19 - 23	04210
7	94	39 - 43	04210
8	94	1 - 5	04220
9	113 bottom	1 - 10	04320
10	114 bottom	1 - 10	04330
11	115	15 , 16 , 17 , 19 , 21	04330
12	115	1 - 6	04330
13	118	1 - 10	04410
14	119	29 - 34	04410
15	119	35 - 40	04410
16	119	1 - 6	04410
17	123	9 , 10 , 18 - 21	04510
18	123	28 - 32	04510
19	124 bottom	1 , 2 , 3 , 5 , 7	04530
20	124 bottom	17 - 20	04530

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card
it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

Volume IV Segment 1 Begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	1 6	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 4	(Volume Number)
	66	and	68	0 1	(Segment Number)

Your READING ASSIGNMENT for this Segment is pg: 86 - 88 .

Now that you have read about the procedure of solving equations by undoing the operations which formed them, you will be asked a series of questions to draw your attention to the more important points.

Question 1

Combine the similar terms and choose the expression equal to

$$5x + 2a - x + 3b$$

(A) $4x + 5ab$

(B) $5 + 2a + 3b$

(C) $4x + 2a + 3b$

(D) $5 + 5ab$

$\frac{2}{1}$

Your work is almost perfect. You seem to have made a mistake in combining two y terms. Shouldn't the result of combining terms with y be another term with y ?

This choice is not correct.

Please return to page $\frac{23}{2}$ and try question 4 again.

$\frac{2}{2}$

The ADDITION PRINCIPLE would be used to undo a subtraction. However, the equation had no subtraction in it.

Therefore, this choice is not correct.

Please return to page $\frac{21}{2}$ and try question 7 again.

In order for terms to be similar the base and exponents must be identical while the coefficients can be different.

For example, in reference to x these are similar:

(1) ax^2 , bx^2

(2) ax , bx

(3) axy , bxy

These are not similar:

(1) ax^2 , ax

(2) ax , a

(3) ax , ay

Combining the similar terms

$3a^2$ and

$-2a^2$ we get

a^2

The

$5a + 4a$ equals

$9a$

Thus we have $a^2 + 9a - 1$

where the three terms are not similar, and must remain separate.

Please proceed to question 3 below.

Question 3

Apply your knowledge to combine terms as far as possible:

$$2xy + 5x^2y + 3x^2 - 2x^2y + 2xy$$

(A) $4xy + 6x^2y$

(B) $7x^2y + 3x^2$

(C) $4xy + 5x^2$

(D) $4xy + 3x^2y + 3x^2$

$\frac{4}{1}$

The ADDITION PRINCIPLE would be used to undo a subtraction, but there was no subtraction in the equation.

Therefore, this choice is not correct.

Return to page $\frac{27}{2}$ and try question 6 again.

$\frac{4}{2}$

When you combine the terms

$$24.6 - 2.4$$

you do get

$$22.2$$

However, you were asked to use the DIVISION PRINCIPLE; therefore, this choice is not correct. You are right if you point out that you are instructed to combine similar terms as the first step in solving an equation. However, this question asks you to apply the DIVISION PROPERTY first, not to solve the equation.

Return to page $\frac{24}{2}$ and try question 9 again.

$\frac{5}{1}$

Since 2.3 was added to the left side of the equation, it is certainly correct to subtract it from both sides in solving the equation. However, before doing that, you should have combined similar terms.

This choice is not correct.

Please return to page $\frac{29}{2}$ and try question 8 again.

$\frac{5}{2}$

If x is used to represent the number, then

$$2 - \frac{1}{2}x$$

would be a translation of

" If one-half a number is subtracted from 2 ... "

This is not in agreement with the problem.

Therefore, this choice is not correct.

Please return to page $\frac{31}{2}$ and try question 11 again.

$\frac{6}{1}$

Similar terms are terms whose literal factors are the same.

You are correct in combining $5x$ and $-x$ and getting $4x$. However, since $2a$ and $3b$ are not similar terms, they cannot be combined.

Therefore, this choice is not correct.

Return to page $\frac{1}{1}$ and try question 1 again.

$\frac{6}{2}$

You have not made a mistake in logic, just in procedure.

It is a proper procedure to get the letter terms alone on one side of the equation, in order to solve the equation, but it is much more sensible to simplify the equation by first combining terms which can be combined.

Since this choice did not do so, it is not correct.

Return to page $\frac{18}{2}$ and try question 5 again.

$\frac{7}{1}$

In order to be able to combine terms, they must be similar.

Although terms such as

$$3x \quad \text{and} \quad 2x^2$$

both contain an x, they are not considered similar terms. To be similar the variable of the base and its exponent must be the same. Therefore, $3x$ is referred to as an x term while $2x^2$ is referred to an an " x squared " term.

Did you think that a term containing x^2 and a term which contains only x are similar?

Since they are not, this choice is not correct.

Please return to page $\frac{23}{2}$ and try question 4 again.

$\frac{7}{2}$

The first operation which you should perform in solving an equation is to combine similar terms. You did a fine job in combining some similar terms, but what about the others?

This choice is not correct.

Please return to page $\frac{20}{2}$ and try question 10 again.

$\frac{8}{1}$

You have apparently become confused about similar terms.

If one term contains

$$a^2$$

and another term contains just

$$a$$

they are not similar and they cannot be combined.

Therefore, this choice is not correct.

Return to page $\frac{19}{2}$ and try question 2 again.

$\frac{8}{2}$

The SUBTRACTION PRINCIPLE would be used to undo an addition.

However, the equation had no addition in it.

Therefore, this choice is not correct.

Return to page $\frac{21}{2}$ and try question 7 again.

The MULTIPLICATION PRINCIPLE would be used to undo a division. But there was no division in the equation. Then this choice is not correct.

Please return to page $\frac{27}{2}$ and try question 6 again.

You did a fine job of dividing each term on the left side of the equation by 6. But what happened to the right side of the equation?

This choice is not correct.

Remember the basic rule: the DIVISION PROPERTY

if, $ax = b$

then, $\frac{ax}{a} = \frac{b}{a}$

or $x = \frac{b}{a}$

Please return to page $\frac{24}{2}$ and try question 9 again.

$\frac{10}{1}$

You are correct in using $5x$ and x as similar terms. However, you have forgotten that each term has a coefficient.

You see, what you have actually said is:

$$ax - x = a$$ and the correct rule is

$$ax - 1x = (a - 1)x$$

What is the coefficient of the term $-x$?

Please return to page $\frac{1}{2}$ and try question 1 again.

$\frac{10}{2}$

It is important to remember that only similar terms can be combined.

A term containing x^2 is not similar to a term which contains

$$x^2y,$$

and they cannot be combined. The variables of the base and their corresponding exponents must be identical in order for the terms to be similar. Therefore, the choice is not correct.

Please return to page $\frac{3}{2}$ and try question 3 again.

In order to get this equation, it is necessary to combine terms on the left side of the equation and also to add 2 to both sides of the equation. But that involves two different operations. Which is the operation that you should perform first? Applying the correct first operation will give you the correct choice.

Please return to page 18/2 and try question 5 again.

You are right in combining similar terms before doing anything else with the equation. But you have made a mistake. What is the coefficient of the term -m ? Yes it is -1 , but where does the decimal point belong in that number? Since the other coefficients contain decimal points, it is necessary that this also contain a decimal point.

Please return to page 29/2 and try question 8 again.

$\frac{12}{1}$

One of the other choices is correct; this is not.

Remember, first locate the similar terms and group them; and then combine the numbers of each set.

Please return to page $\frac{19}{2}$ and try question 2 again.

$\frac{12}{2}$

If we represent the number by x the problem is translated as

$$\frac{1}{2}x - 2 = 9$$

Then, in solving the equation, we use the ADDITION PRINCIPLE to undo the subtraction of 2 . This does not give us the equation offered in this choice.

Please return to page $\frac{31}{2}$ and try question 11 again.

$\frac{13}{1}$

In order to combine terms, you must convert all of the expression into individual terms. Therefore, it is first necessary to get rid of the parentheses. This requires that you apply the DISTRIBUTIVE PROPERTY. The rule to remember is:

$$a(b + c) = ab + ac$$

Did you forget to distribute the multiplier each time?

This choice is not correct.

Please return to page $\frac{23}{2}$ and try question 4 again.

$\frac{13}{2}$

The left side of the equation is one quantity, even though it is made up of three terms. Consequently, it is necessary to apply the DISTRIBUTIVE PROPERTY when dividing this quantity by 6 .

That is $a + b + c = d$ when divided by k

becomes $(a + b + c) \div k = d \div k$ by applying the DISTRIBUTIVE PROPERTY

this becomes $\frac{a}{k} + \frac{b}{k} + \frac{c}{k} = \frac{d}{k}$

Since you forgot the DISTRIBUTIVE PROPERTY, this choice is not correct.

Please return to page $\frac{24}{2}$ and try question 9 again.

14/1

The equation, which is in the form

$$\frac{ax}{b} = c$$

contains the operations of multiplication and division. Undoing them in reverse order from that in which they would be performed, we should first undo the division, but this requires that we use the MULTIPLICATION PRINCIPLE. Then this choice is not correct.

Please return to page 21/2 and try question 7 again.

14/2

You should always check a solution to detect a possible error in your method or in your computation.

If we substitute 4 for x we get:

$$5(4) + 17 - 4 \stackrel{?}{=} 35$$
$$20 + 17 - 4 \stackrel{?}{=} 35$$
$$37 - 4 \stackrel{?}{=} 35$$
$$33 \neq 35$$

NOTE: $\stackrel{??}{=}$ means "does it equal?"

Since this does not "check" this choice is not correct.

Please return to page 36/2 and try question 12 again.

$\frac{15}{1}$

Every term has a coefficient which has to be used when combining similar terms. What is the coefficient of $-x$?

You also have forgotten that only similar terms can be combined,

$$2a \quad \text{and} \quad 3b$$

are not similar terms.

This choice is not correct.

Please return to page $\frac{1}{2}$ and try question 1 again.

$\frac{15}{2}$

It is important to remember that only similar terms can be combined. Terms containing

$$xy \quad \text{and} \quad x^2y$$

are not similar and they cannot be combined. The variables of the base and their corresponding exponents must be identical in order for the terms to be similar.

Therefore, this choice is not correct.

Please return to page $\frac{3}{2}$ and try question 3 again.

16/1

Since the equation, which is in the form

$$ax + b = c$$

has two operations in it, multiplication and addition, it is necessary to apply the inverse operations to solve the equation. In performing operations, multiplication is done before addition. Therefore, in undoing the operation, we undo the addition first, and then the multiplication. Since division is the inverse of multiplication, it is the second principle to be used.

Then this choice is not correct.

Please return to page 27/2 and try question 6 again.

16/2

The first operation which you should perform in solving an equation is to combine similar terms. Since you have not done that, this choice is not correct.

The principle which you have applied is a correct one, but it should not have been used until later.

Please return to page 20/2 and try question 10 again.

It is correct to combine terms first when solving an equation.

However, it is only possible to combine similar terms.

A term containing a variable of the first degree cannot be similar to a constant. Therefore, they cannot be combined.

$$ax \quad \text{and} \quad a$$

for example cannot be combined into one term and must remain as

$$ax + a$$

two terms, a binomial

This choice is not correct.

Please return to page 29/2 and try question 8 again.

If we represent the number by x, the problem is translated as

$$\frac{1}{2}x - 2 = 9$$

Since the x has been multiplied by $\frac{1}{2}$, when we wish to undo it, we must divide by $\frac{1}{2}$. But this choice shows that there has been a multiplication by $\frac{1}{2}$. Besides, the 9 and the 2 should be combined into 11 before any attempt is made to use the DIVISION PROPERTY.

This choice is not correct.

Please return to page 31/2 and try question 11 again.

$\frac{18}{1}$

In order to combine similar terms, it is first necessary to get rid of the parentheses. Then we have:

$$3(x + 4y) + 5(x - y) + 2x^2 =$$
$$3x + 12y + 5x - 5y + 2x^2 =$$
$$8x + 7y + 2x^2$$

Therefore, this choice is correct.

Please proceed to question 5 below.

$\frac{18}{2}$

Question 5

In solving the equation

$$5m + 3m - 2 = 14$$

choose the derived equation which should be the result of the first operation that is recommended.

(A) $5m + 3m - 2 + 2 = 14 + 2$

(B) $8m - 2 = 14$

(C) $8m = 16$

(D) $m = 2$

Since

$$5x \quad \text{and} \quad -x$$

are similar terms, they can be combined. The coefficient of

$$-x \quad \text{is} \quad 1$$

so that we get

$$4x$$

The remaining terms are not similar, and they cannot be combined.

Therefore, this choice is correct.

In summary unless the terms are of the form:

$$ax \pm bx$$

that is

$$ax + bx = (a + b)x$$

or

$$ax - bx = (a - b)x$$

they can't be combined.

Proceed to question 2 below.

Question 2

Combine the similar terms and choose the expression equal to

$$3a^2 + 5a + 4a - 2a^2 - 1$$

(A) $a^2 + 9a - 1$

(B) $10^2 - 1$

(C) $10^3 - 1$

(D) None of these.

$\frac{20}{1}$

When dividing the left side of the equation by 6 the DIVISION PRINCIPLE for equations tells you to divide the right side by 6 also. But the left side contains three terms. Then it is necessary to apply the DISTRIBUTIVE PRINCIPLE and to divide each term by the 6 . Since you have done that, the equation you have obtained is correct. Note that it is not the preferred procedure you would use in solving the equation because you would first combine similar terms. However, it does produce a legitimate derived equation which will lead to the correct solution.

Please proceed to question 10 below.

$\frac{20}{2}$

Question 10

For the equation

$$31.2 + 5.4 = 17 + 1.2x - .3x$$

apply the recommended procedure to find the first derived equation in solving for x .

(A) $36.6 = 17 + 1.2x - .3x$

(B) $31.2 + 5.4 = 17 + .9x$

(C) $31.2 + 5.4 - 17 = 1.2x - .3x$

(D) $36.6 = 17 + .9x$

Since the equation has two operations in it, multiplication and addition, it is necessary to apply the inverse operations to solve the equation. In performing operations, multiplication is done before addition.

Therefore, in undoing the operations, we undo the addition first.

That is,

$$\begin{aligned} ax + b &= c \\ -b &= -b \\ ax &= c - b \end{aligned}$$

Consequently, the first operation to be performed is subtraction, and this choice is correct.

Please proceed to question 7 below.

Question 7

In solving the equation

$$\frac{2}{3}n = 21$$

choose which of the following transformation principles should be used first.

(A) Addition

(B) Subtraction

(C) Multiplication

(D) Division

$\frac{22}{1}$

If you follow in order all the steps necessary to solve the equation, you will get this result. However, you were asked to find the equation which would be the derived equation resulting from the first recommended operation.
The equation you have chosen is actually the fourth step in the solution.

Please return to page $\frac{18}{2}$ and try question 5 again.

$\frac{22}{2}$

In solving this equation, you should proceed as follows:

First; combine similar terms on both sides of the equation.
Second: apply the ADDITION PRINCIPLE to get the y term alone on one side of the equation.
Third: apply the DIVISION PRINCIPLE to remove the coefficient of the y term.

When this is done, you will discover that the solution is not in the set offered in this choice.

Please return to page $\frac{43}{2}$ and try question 13 again.

It is important to remember that only similar terms can be combined.

The two terms containing

$$xy$$

combine to form $4xy$

The two terms containing

$$x^2y$$

combine to give $3x^2y$

The terms which contain just

$$x^2$$

are unlike any other and do not combine with any of them.

Therefore, this answer is correct.

Please proceed to question 4 below.

Question 4

Apply your knowledge to combine similar terms as far as possible:

$$3(x + 4y) + 5(x - y) + 2x^2$$

(A) $8x + 7y + 2x^2$

(B) $8x + 7 + 2x^2$

(C) $10x^2 + 7y$

(D) $8x + 3y + 2x^2$

$\frac{24}{1}$

The left side of this equation contained four terms of which three were similar.

Then the first step should be to combine the three terms.

Combining

$$5.2m + 6.3m - 1m$$

does give us $10.5m$

Then this choice is correct.

Of course, it is possible to proceed differently, but it is not wise.

Whenever you can, you should gather separate items into one package before dealing with them.

Please proceed to question 9 below.

$\frac{24}{2}$

Question 9

As an alternate exercise, apply the DIVISION PROPERTY to the equation:

$$6y + 24.6 - 2.4 = 12$$

to solve for y and select the result of this operation.

(A) $6y + 22.2 = 12$

(B) $y + 4.1 - .4 = 12$

(C) $y + 24.6 - 2.4 = 2$

(D) $y + 4.1 - .4 = 2$

You should always check a solution to detect a possible error in your method or in your computation.

If we substitute 4.4 for x , we get:

$$5(4.4) + 17 - 4.4 \stackrel{?}{=} 35$$

NOTE: $\stackrel{?}{=}$ means " does it equal? "

$$22.0 + 17 - 4.4 \stackrel{?}{=} 35$$

$$39 - 4.4 \stackrel{?}{=} 35$$

$$34.6 \neq 35$$

Even though this is close to 35 , it is not equal to 35 .

Therefore, this choice is not correct.

Please return to page $\frac{36}{2}$ and try question 12 again.

If we substitute 5 for p we get:

Given: $5p + 3 + 2p = 5$

$$5(5) + 3 + 2(5) \stackrel{?}{=} 5$$

$$25 + 3 + 10 \stackrel{?}{=} 5$$

$$38 \neq 5$$

Since 38 does not equal 5 this choice is not correct.

Please return to page $\frac{39}{2}$ and try question 15 again.

$\frac{26}{1}$

In solving this equation, you should proceed as follows:

First: combine similar terms on both sides of the equation.

Second: apply the SUBTRACTION PRINCIPLE to get the term containing t alone on one side of the equation.

Third: apply the DIVISION PRINCIPLE to remove the coefficient of the t term.

When you follow these steps correctly, you will find that this choice is not correct.

Please return to page $\frac{33}{2}$ and try question 14 again.

$\frac{26}{2}$

In order to solve this equation, proceed as follows:

First: combine similar terms.

Second: apply the ADDITION PRINCIPLE to get the x term alone on one side.

Third: apply the DIVISION PRINCIPLE to remove the coefficient of x .

When you complete these steps, you will discover that your solution does not agree with this choice.

Please return to page $\frac{32}{2}$ and try question 17 again.

The first step in the solution of an equation should be to combine terms on each side so as to simplify the equation.

If we do that, we get exactly the equation offered in this choice.

Then this choice is correct.

Please proceed to question 6 below.

Question 6

In solving the equation

$$6x + 2 = 5$$

choose which of the following transformation principles should be used first.

(A) Addition

(B) Subtraction

(C) Multiplication

(D) Division

28/1

You should always check a solution to detect a possible error in your method or in your computation.

If we substitute 4.8 for x , we get:

$$5(4.8) + 17 - 4.8 \stackrel{?}{=} 35$$
$$24.0 + 17 - 4.8 \stackrel{?}{=} 35$$
$$41.0 - 4.8 \stackrel{?}{=} 35$$
$$36.2 \stackrel{?}{=} 35$$

Note: $\stackrel{?}{=}$ means "does it equal?"

Since this does not equal 35 this choice is not correct.

Please return to page 36/2 and try question 12 again.

28/2

In order to solve the equation, you should proceed as follows:

First: combine similar terms on both sides.

Second: apply the SUBTRACTION PRINCIPLE to get the term with t alone on one side.

Third: apply the DIVISION PRINCIPLE to remove the coefficient of t .

When you perform these steps, your solution will not agree with this choice.

Please return to page 45/2 and try question 16 again.

The equation contains a multiplication (by 2) and a division (by 3). Following the principle that we undo the operations in the reverse order from that in which they would be performed, the first operation to undo is the division, and therefore, we use the MULTIPLICATION PRINCIPLE getting

$$2n = 63$$

While it is possible to use the DIVISION PRINCIPLE first, it is not wise since that would give us

$$\frac{1}{3}n = \frac{21}{2}$$

Most students would agree that they prefer to avoid fractions when possible, and it certainly is avoidable in this case. Observe that if you apply the operations in <u>reverse</u> order when you solve an equation, you will get the easiest calculation as a result.

This choice is correct.

Please proceed to question 8 below.

<u>Question 8</u>

For the equation

$$5.2m + 2.3 + 6.3m - m = 23.3$$

choose the derived equation which is the result of the first operation in the solution.

(A) $10.5m + 2.3 = 23.3$

(B) $5.2m + 6.3m - m = 23.3 - 2.3$

(C) $11.4m + 2.3 = 23.3$

(D) $12.8m + 23.3$

$\frac{30}{1}$

If we substitute

$$3\frac{1}{2} = \frac{7}{2}$$

for p, we get:

Given: $5p + 3 + 2p = 5$

$5(\frac{7}{2}) + 3 + 2(\frac{7}{2}) \stackrel{?}{=} 5$

$\frac{35}{2} + 3 + 7 \stackrel{?}{=} 5$

$17.5 + 10 \stackrel{?}{=} 5$

$27.5 \neq 5$

Since 27.5 does not equal 5 this choice is not correct.

Please return to page $\frac{39}{2}$ and try question 15 again.

$\frac{30}{2}$

In order to solve this equation, proceed as follows:

First: combine similar terms

Second: apply the SUBTRACTION PRINCIPLE to get the m term alone on one side.

Third: apply the MULTIPLICATION PRINCIPLE to remove the fraction from the left side.

Fourth: apply the division principle to remove the coefficient of m .

When you complete these steps, you will discover that your solution does not agree with this choice.

Please return to page $\frac{37}{2}$ and try question 18 again.

$\frac{31}{1}$

The first operation you should perform in solving an equation is to combine similar terms. Since you have done that completely and correctly, this choice is correct.

Please proceed to question 11 below.

$\frac{31}{2}$

Question 11

If 2 is subtracted from one-half a number, the result is 9. If an equation is written to solve this problem, apply your knowledge to find the first derived equation in the solution of the equation.

(A) $\frac{1}{2}x = 11$

(B) $2 - \frac{1}{2}x = 9$

(C) $\frac{1}{2}x = 7$

(D) $x = \frac{1}{2}(9) + 2$

$\frac{32}{1}$

In order to solve this equation, you should proceed as follows:

Given: $15 + 4 - 1 = 8 + 3t - t$	$[C \wedge A$	Combining similar terms
$18 = 8 + 2t$	$\propto -8$	Subtracting 8 from both sides
$10 = 2t$	$\propto \div 2$	Dividing by 2 on both sides
$5 = t$		

Notice that if

$$5 = t$$

it follows from the SYMMETRIC PROPERTY that

$$t = 5$$

Since 5 is between 4 and 6 , it follows that this choice is correct.

Please proceed to question 17 below.

$\frac{32}{2}$

Question 17

Apply your knowledge to find the correct statement about the solution of the equation

$$5x + 7x - 2 + 2x = 12$$

(A) $x \in \{\text{even integers}\}$

(B) $x \in \{\text{odd integers}\}$

(C) $x \in \{\text{proper fractions}\}$

(D) $x \in \{\text{improper fraction}\}$

In order to solve this equation, we use the following procedure:

Given: $3y + 5y - 2 = 36 - 6$ [C ^ A

$8y - 2 = 30$ $\propto +2$

$8y = 32$ $\propto \div 8$

$y = 4$

Note: The symbol [denotes a principle that applies to terms. The symbol $\propto$ denotes an operation that applies to both sides of the equation. In general it indicates the INVERSE OPERATION of the one involving the quantity that is to be eliminated.

Since your choice was an element of the set you picked, your answer is correct.

Please proceed to question 14 below.

Question 14

Apply your knowledge to find the true statement about the solution of the equation

$$10t - 3 + 4t + 19 - 3t = 25$$

(A) $t = 1$

(B) $t < 1$

(C) $t > 1$

(D) None of these.

$\frac{34}{1}$

Since the variable appears on both sides of the equation, you should begin by using the SUBTRACTION PROPERTY OF EQUALITY to remove the variable from one side. Write the equivalent equation, resulting from applying the operation:

$$\propto -2x$$

to the equation.

Please return to page $\frac{40}{2}$ and try this question again.

$\frac{34}{2}$

We don't agree. After simplifying the expression with the parentheses, we get

$$3 + 8r + 4 = 18 + 3r$$

Now collect like terms.

To obtain an equivalent equation with the variables on the left side, subtract $3r$ from both members of the equation. Please continue.

Please return to page $\frac{60}{2}$ and try this question again.

If you apply the proper principles to solve the equation, you will find that one of the other choices is correct.

Please return to page 39/2 and try question 15 again.

Using x to represent the number, we translate the problem as:

$$2x - 5 + 9 = 88$$

Then, combining terms is the next step, but it does not give this choice as the result. Therefore, this choice is not correct.

Note that 5 less than twice a number is

$$2x - 5$$

not

$$5 - 2x$$

Please return to page 42/2 and try question 19 again.

$\frac{36}{1}$

If we represent the number by x, the problem is translated as

$$\frac{1}{2}x - 2 = 9$$

Then this equation contains a division (by 2) and a subtraction (of 2). Undoing the operations in reverse order, we first take care of the subtraction by adding 2 to both sides of the equation. Then we get the equation offered in this choice, so that this choice is correct.

We might notice that $\frac{1}{2}x$

can be considered as x divided by 2 or as x multiplied by $\frac{1}{2}$; it has no effect on the result, but there is a slight difference in the thinking involved. For example:

if $\frac{x}{a} = b$	while if $(\frac{1}{a})x = b$	
then we multiply by a	we divide by $(\frac{1}{a})$	Since: division by a quantity is the same as multiplication by by the inverse of the quantity. $\div (\frac{1}{a}) = x\,(a)$
$a\frac{x}{a} = ab$	$\frac{1}{a}x \div \frac{1}{a} = b \div \frac{1}{a}$	
$x = ab$	$\frac{1}{a}x \cdot a = ab$	
	$x = ab$	

Please proceed to question 12 below.

$\frac{36}{2}$

Question 12

Perform the calculation to find which choice is the solution of the equation

$$5x + 17 - x = 35$$

(A) $x = 4$ (C) $x = 4.5$

(B) $x = 4.4$ (D) $x = 4.8$

In order to solve this equation proceed as follows:

$$5x + 7x - 2 + 2x = 12 \qquad [C \wedge A$$

$$14x - 2 = 12 \qquad \propto + 2$$

$$14x = 14 \qquad \propto \div 14$$

$$x = 1$$

Since this value is an element of the set of odd integers, this choice is correct.

Please proceed to question 18 below.

Question 18

Apply your knowledge to find the correct statement about the solution of the equation

$$\frac{3}{5}m + \frac{1}{4} + \frac{4}{5}m = 2$$

(A) $m \in \{\text{even integers}\}$

(B) $m \in \{\text{odd integers}\}$

(C) $m \in \{\text{proper fractions}\}$

(D) $m \in \{\text{improper fractions}\}$

$\frac{38}{1}$

If w represents William's age,

$$2w + 5$$

represents Bob's age. Then the correct translation of the problem is

$$w + 2w + 5 = 50$$

Now, the next step in the solution of this equation is to combine similar terms. But this does not give the equation of this choice.

Therefore, this choice is not correct.

Please return to page $\frac{56}{2}$ and try question 20 again.

$\frac{38}{2}$

You made a careless error. Before the third step, you obtained the equation

$$7y + 10 = 17$$

You should now use the SUBTRACTION PROPERTY OF EQUALITY to eliminate the 10

Please return to page $\frac{55}{2}$ and try this question again.

In order to solve this equation, we proceed as follows:

Given:	$10t - 3 + 4t + 19 - 3t = 25$	[C^ A	(Combining similar terms)
	$11t + 16 = 25$	∝< - 16	(Subtracting 16 from both sides)
	$11t = 9$	∝< ÷ 11	(Dividing by 11 on both sides)
	$t = \frac{9}{11}$		

Then the correct value of t is less than 1, and this choice is correct.

Please proceed to question 15 below.

Question 15

Perform the calculation to find which value is the solution of the equation

$$5p + 3 + 2p = 5$$

(A) 5

(B) $3\frac{1}{2}$

(C) $\frac{2}{7}$

(D) None of these.

40
1

Volume 4 Segment 2 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS:	48	and	50	1 7	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 4	(Volume Number)
	66	and	68	0 2	(Segment Number)

Your READING ASSIGNMENT for this segment is pages 91 to 93.

SUPPLEMENTARY NOTES:

In the previous segment, you became familiar with a method for solving linear equations in one variable where the first operation to perform was to collect like terms. This was then followed by an application of the ADDITION and SUBTRACTION PROPERTIES and the MULTIPLICATION and DIVISION PROPERTIES.

Now, we will consider equations where the variable is found on both sides of the equation.

You will now be asked a series of questions to draw your attention to the more important points.

40
2

Perform the first two transformations that are necessary to simplify the equation and select the letter next to the resulting derived equation

$$5x - 8 = 2x + 3$$

(A) $3x = 5$ (B) $3x = 11$

(C) $7x = 11$ (D) $\frac{5}{2}x = \frac{3}{8}$

$\frac{41}{1}$

Please note that

$$4(2r + 1) =$$

$$8r + 4$$

The number immediately before a parentheses multiplies each number within the parenthesis.

This is an example of the DISTRIBUTIVE LAW

$$a(b + c) = ab + ac$$

Please return to page $\frac{60}{2}$ and try this question again.

$\frac{41}{2}$

We don't agree. Did you begin by simplifying the expression with the parentheses? That should be the first step.

Please note that

$3(5 + 2s) = 15 + 6s$

and $13(s + 3) = 13s + 39$ by applying [D (the DISTRIBUTIVE LAW)

Please return to page $\frac{50}{2}$ and try this question again.

42
1

In order to solve this equation proceed as follows:

Given: $\frac{4}{5}m + \frac{3}{5}m + \frac{1}{4} = 2$ [C ^ A

$\frac{7}{5}m + \frac{1}{4} = 2$ ≺ $-\frac{1}{4}$

$\frac{7}{5}m = \frac{7}{4}$ ≺ $\cdot 5$

$7m = \frac{35}{4}$ ≺ $\div 7$

$m = \frac{5}{4}$

Then m is an improper fraction and this choice is correct.

If you find that the arithmetic with fractions is very difficult for you, it will be necessary to ask your instructor for some special instruction on fractions.

Please proceed to question 19 below.

42
2

Question 19

If 5 less than twice a number were increased by 9 , the result would be 88 . If you write an equation to solve this problem, apply your knowledge to find the derived equation resulting from the first operation performed.

(A) $14 - 2x = 88$

(B) $2x + 4 = 88$

(C) $5 - 2x = 88$

(D) $2x - 5 = 79$

Good, we should always check a solution to detect a possible error in method or computation.

Given:	$5x + 17 - x = 35$	Let	$x = 4.5$
	$5(4.5) + 17 - (4.5) \stackrel{?}{=} 35$	Note:	The symbol $\stackrel{?}{=}$
	$22.5 + 17 - 4.5 \stackrel{?}{=} 35$	means	" does it equal? "
	$39.5 - 4.5 \stackrel{?}{=} 35$		
	$35 = 35$		

To solve the original equation, we should use the following procedure and format.

Given:	$5x + 17 - x = 35$ $[C \wedge A$	Use the COMMUTATIVE and ASSOCIATIVE PROPERTIES to combine like terms. Use the SUBTRACTION PROPERTY OF EQUALITY and subtract 17 from both sides of the equation.
	$4x + 17 = 35 \prec - 17$	
	$4x = 18 \prec \div 4$	Use the DIVISION PROPERTY OF EQUALITY and divide both sides of the equation by 4 .
	$x = 4.5$	

Please proceed to question 13 below.

Question 13

Apply your knowledge to solve the equation

$$3y + 5y - 2 = 36 - 6$$

(A) $\{0 , 1 , 2\}$ (C) $\{1 , 3 , 5\}$

(B) $\{1 , 2 , 3\}$ (D) $\{4 , 6 , 8\}$

$\frac{44}{1}$

We don't agree.

The given equation is

$$y + 8y + 10 = 2y + 17$$

Start by combining the like terms on the left side of the equation.

You will get:

$$9y + 10 = 2y + 17$$

What property of equality should you use to have the variables appear only on the left side of the equation?

Consider $\propto - 2y$

Please return to page $\frac{55}{2}$ and try this question again.

$\frac{44}{2}$

To obtain the solution set of an equation, we first isolate the variable on the left side of the equation.

Since the variable also appears on the right side, it is necessary to apply the property of equality which will remove it from the right side.

What will accomplish this?

Please return to page $\frac{57}{2}$ and try this question again.

If we substitute $\frac{2}{7}$ for p , we get:

Given:	5p	+	3	+	2p	=	5	
	$5\left(\frac{2}{7}\right)$	+	3	+	$2\left(\frac{2}{7}\right)$	$\stackrel{?}{=}$	5	
	$\frac{10}{7}$	+	3	+	$\frac{4}{7}$	$\stackrel{?}{=}$	5	
	$\frac{14}{7}$	+	3			$\stackrel{?}{=}$	5	
	2	+	3			$\stackrel{?}{=}$	5	
			5			=	5	

Then this choice is correct.

It would be more efficient to solve the equation:

	5p	+	3	+	2p	=	5	[C^A (Combining similar terms)
	7p	+	3			=	5	∝- 3 (Subtracting 3 from both sides)
	7p					=	2	∝÷ 7 (Dividing by 7 on both sides)
	p					=	$\frac{2}{7}$	

Please proceed to question 16 below.

Question 16

Apply your knowledge to find the true statement about the solution of the equation

$$15 + 4 - 1 = 8 + 3t - t$$

(A) $t < 2$

(B) $2 < t < 4$

(C) $4 < t < 6$

(D) $t > 6$

$\frac{46}{1}$

Well it looks like you handled the problem of the $5x$ and $2x$ being on opposite sides of the equation without any difficulty. But a similar problem presented itself with the -8 and the $+3$ also on opposite sides. Don't you agree that $\propto +8$ would be a good method of taking care of that?

Please return to page $\frac{40}{2}$ and reconsider the problem.

$\frac{46}{2}$

We don't agree.

Let us do a similar question together. Find the solution set of the equation

$7x$	-28	$=$	$3x$ $\propto -3x$	To get the variables on the left side of the equation, we have to add the inverse of $3x$ to both members of the equation.
$4x$	-28	$=$	0 $\propto +28$	Next, we add 28 to both sides.
$4x$		$=$	28 $\propto \div 4$	Finally, we divide by the coefficient of x
$\frac{4x}{4}$		$=$	$\frac{28}{4}$	
x		$=$	7	

Please return to page $\frac{62}{2}$ and try this question again.

Using x to represent the number, we translate the problem as

$$2x - 5 + 9 = 88$$

Did you forget that 5 less than twice a number is

$$2x - 5 \text{ not } 5 - 2x?$$

This choice is not correct.

Please return to page 42/2 and try question 19 again.

We don't agree with your choice. How did you start this question? The first step is to write a derived equation with the variables appearing only on one side. You accomplish this by using the SUBTRACTION PROPERTY OF EQUALITY and subtract 4m from both members of the given equation. Then you should eliminate the +4 , and finally eliminate the coefficient of the variable m .
In each operation be careful to choose the "inverse" of the quantity that you are attempting to eliminate.

Before you are satisfied with your choice, you should check your answer by substitution in the original equation.

Please return to page 54/2 and try this question again.

If w represents William's age,

$$2w + 5$$

represents Bob's age. Then the correct translation of the problem is

$$w + 2w + 5 = 50$$

Now the next step in the solution of this equation is to combine similar terms by using the COMMUTATIVE and ASSOCIATIVE LAWS, symbolized by

$$[\ C\ \wedge\ A$$

giving us $3w + 5 = 50$

Therefore, this choice is correct.

You have now finished Segment 1 . Hand in the PUNCH CARD.
You should enter in your NOTEBOOK the following:

In solving equations, follow these steps:

(1) Combine similar terms $[\ C\ \wedge\ A$

(2) Apply the ADDITION or SUBTRACTION PRINCIPLES to get only terms with the variable on one side, and only terms without the variable on the other side.

e.g. $x - a = b \propto +a$ | $x + a = b \propto -a$

$x = b + a$ | $x = b - a$

(3) Apply the MULTIPLICATION PRINCIPLE to remove any fraction from the coefficient of the variable.

e.g. $\frac{x}{a} = b \propto \cdot a$

$x = ab$

(4) Apply the DIVISION PRINCIPLE to remove the coefficient of the variable.

e.g. $ax = b \propto \div a$

$x = \frac{b}{a}$

You should now be able to complete HOMEWORK ASSIGNMENT Number 4 , problems 1-4

You made a hasty decision.

One of the letters does have the correct answer next to it.

Please return to page $\frac{55}{2}$ and try this question again.

To eliminate a term, we choose the additive inverse of that term and apply the ADDITION OR SUBTRACTION PROPERTY.

You used the wrong property of equality. The inverse of

$+ 2x$ is $- 2x$

Please return to page $\frac{57}{2}$ and try this question again.

50
1

Very good. You made the correct choice. What procedure do we follow? When an equation contains parentheses, begin by simplifying these expressions.

Thus,	$3 + 4(2r + 1) = 18 + 3r$	[D	this means that we will use the DISTRIBUTIVE PROPERTY.
	$3 + 8r + 4 = 18 + 3r$	[C ^ A	Collect
	$8r + 7 = 18 + 3r$	$\propto -3r$	Subtract 3r from both members of the equation.
we get:	$5r + 7 = 18$	$\propto -7$	Subtract 7 from both members of the equation
	$5r = 11$		

Please go on to question 4 below.

50
2

Question 4

Perform the necessary transformations and select the letter next to the equation that is equivalent to the equation

$$12s + 3(5 + 2s) = 13(s + 3).$$

(A) $5s = 1$

(B) $4s = 24$

(C) $4s = 8$

(D) $5s = 24$

Using x to represent the number, we translate the problem as

$$2x - 5 + 9 = 88$$

Then the next step should be to combine similar terms, but that is not what you did. Your procedure is foolish, not incorrect. But if we are to learn to do an efficient job of solving equations, we should perform operations in the best possible order.

Therefore, this choice is not correct.

Please return to page 42/2 and try question 19 again.

Your choice is incorrect.

It appears that you're a bit confused.

Please study the example on page 91 of your reading assignment.

Please return to page 40/2 and try this question again.

$\frac{52}{1}$

By this time you should understand the routine that should be followed to solve equations with unknowns on both sides of the equals sign.

Let's review once more:

First: Apply the ADDITION or SUBTRACTION PROPERTY to remove the variables from one side (it doesn't matter which) and assemble them on the other side.

Second: Remove any constant terms from the side that now has the variable by applying the inverse and using the ADDITION or SUBTRACTION PROPERTY again.

Finally: Remove the coefficient of the variable by choosing its inverse and apply the MULTIPLICATION or DIVISION PROPERTY. This value then, of course, should be checked by substituting it for the variable in the original equation.

Please return to page $\frac{65}{2}$ and tackle this problem again.

$\frac{52}{2}$

This problem might seem complicated, but really it is very similar to the others that you have just done. The only new feature is the additional set of parentheses. Once they have been eliminated by the application of the DISTRIBUTIVE LAW, you should be back in familiar surroundings. You will collect like terms, then maneuver the variables to one side and the constants to the other until you find the value of the variable, which you check.

Please return to page $\frac{76}{2}$ and try this question again.

We do not agree. One of the letters does have the correct answer next to it.

Remember, transform the equation so that the unknowns are on the same side. Then eliminate the coefficients by using the DIVISION PROPERTY.

Return to page 57/2 and try this question again.

After clearing parentheses, the given equation becomes

$$12 + 16 - 8y = 6y$$

Observe that the variables are on different sides of the equality sign. Furthermore, by the RELEXIVE PROPERTY OF EQUALITY,

(Remember? if $a = b$
then $b = a$) . . . . the equation can be written,

$$6y = 12 + 16 - 8y$$

Please continue.

Return to page 59/2 and try this question again.

$\frac{54}{1}$

Very good. You made the correct choice. We have to find the solution set of the equation

$$9z - 24 = 3z$$

Our first objective is to obtain an equivalent equation with the variables appearing only on one side of the equation.

$9z - 24 = 3z$ $\quad \swarrow -3z$ Subtracting $3z$ from both members,

$6z - 24 = 0$ $\quad \swarrow +24$ Now, we add the inverse of -24, namely, $+24$ to both sides,

$6z = 24$ $\quad \swarrow \div 6$ Dividing both members of the equation by the coefficient of the variable,

$z = 4$

which is the solution set. This should be checked:

$$9(4) - 24 \stackrel{?}{=} 3(4)$$

$$36 - 24 \stackrel{?}{=} 12$$

$$12 = 12$$

Please go on to question 7 below.

$\frac{54}{2}$

Question 7

Apply the proper principles and choose the set that contains the solution set of the equation as a subset.

$$8m + 4 = 4m + 16$$

Select the letter next to the correct answer.

(A) $\{12, 14, 16\}$ (C) $\{4, 5, 6\}$

(B) $\{7, 8, 9\}$ (D) $\{1, 2, 3\}$

Very good. You made the correct choice. Since the variable appears on both sides of the equation, you begin by subtracting $2x$ from both members of the equation. Thus,

$$5x - 8 = 2x + 3 \quad \cancel{<}\; - 2x$$

$$3x - 8 = 3 \quad \cancel{<}\; + 8$$

$$3x = 11$$

Next, we use the ADDITION PROPERTY of EQUALITY to eliminate the 8

Now, if we were asked to complete the problem we would eliminate the 3 by using the DIVISION PROPERTY:

$$3x = 11 \quad \cancel{<}\; \div 3$$

$$x = 3\frac{2}{3}$$

Please go on to question 2 below.

Question 2

Perform the first three transformations that are necessary to simplify and select the letter next to the derived equation.

$$y + 8y + 10 = 2y + 17$$

(A) $7y = 27$

(B) $11y = 7$

(C) $7y = 7$

(D) None of these.

56
1

Using x to represent the number, we translate the problem as

$$2x - 5 + 9 = 88$$

Then the next step is combining terms, which gives us

$$2x + 4 = 88$$

Therefore, this choice is correct.

Please proceed to question 20 below.

56
2

Question 20

Bob's age is 5 years more than twice William's age. If the sum of their ages is 50 and w represents William's age, apply your knowledge to find the first derived equation in the solution of the equation which represents the conditions of the problem.

(A) $2w = 45$

(B) $3w = 55$

(C) $3w + 5 = 50$

(D) $2w = 55$

Very good. You made the correct choice. The first step is to simplify the expression with the parentheses. Thus, we have

$12s + 3(5 + 2s) = 13(s + 3)$	[D		
$12s + 15 + 6s = 13s + 39$	[C ^ A		Next we combine like terms,
$18s + 15 = 13s + 39$	<- $13s$		To get the variables on the left side of the equation, we subtract $13s$ from both members,
$5s + 15 = 39$	<- 15		Finally, we subtract 15 from both sides and obtain
$5s = 24$			

Please go on to question 5 below.

Question 5

Apply your knowledge and find the solution set of the equation

$$7x = 45 + 2x$$

Select the letter next to the correct answer.

(A) $\{8\}$

(B) $\{5\}$

(C) $\{9\}$

(D) None of these.

58/1

If you examine this equation, you will see that the first operation is to eliminate the parentheses. Of course, you realize that this calls for the application of the DISTRIBUTIVE LAW.

$$a (b + c) = ab + ac$$

Now, this is true even if " a " happens to be a fraction.

Once this has been accomplished then you will find that you have to collect like terms. From this point on it is just like the problems that you have been doing.

Of course, you must always check your result by substitution. How else will you know whether you are right or wrong? You won't always have us to point out your mistakes.

Now return to page 68/2 and do this problem correctly.

58/2

How do we simplify an expression with more than one symbol of inclusion? Recall that the expression within the innermost symbol is simplified first. This is merely a repeated application of the DISTRIBUTIVE LAW.

$$2 [2 + 3 (x + 4)] \quad [D$$

$$2 [2 + 3x + 12] \quad [C \wedge A$$

$$2 [14 + 3x] \quad [D$$

$$28 + 6x$$

Please return to page 66/2 and continue the solution.

Very good. You made the correct choice. We have to find the solution set of the equation

$$12r - 3 = 4 - 2r$$

$12r - 3 = 4 - 2r$	$\times + 2r$	We begin by adding $2r$ to both members of the equation.
$14r - 3 = 4$	$\times + 3$	We add 3 to both members of the equation.
$14r = 7$	$\times \div 14$	Dividing by the coefficient, 14.
$r = \frac{7}{14}$		
$r = \frac{1}{2}$		

Please go on to question 9 below.

Question 9

Apply your knowledge and find the solution set of the equation,

$$12 + 8(2 - y) = 6y$$

Select the letter next to the correct answer.

(A) $\{14\}$

(B) $\{2\}$

(C) $\{\frac{11}{7}\}$

(D) None of these.

$\frac{60}{1}$

Very good. You made the correct choice. We have the equation

$$y + 8y + 10 = 2y + 17 \quad [\ C \wedge A$$

After combining like terms, we get

$$9y + 10 = 2y + 17 \quad \prec - 2y$$

Next, we subtract $2y$ from both members of the equation,

$$7y + 10 = 17 \quad \prec - 10$$

Finally, 10 is subtracted from both members of the equation, and we obtain

$$7y = 7$$

which is your choice.

Of course, to complete the process of finding the value for y we divide both sides of the equation by 7.

$$7y = 7 \quad \prec \div 7$$

$$y = 1$$

which we should verify by substitution.

Please go on to question 4 below.

$\frac{60}{2}$

Question 4

Perform the necessary transformations and select the letter next to the equation that is derived from the equation

$$3 + 4(2r + 1) = 18 + 3r$$

(A) $5r = 11$

(B) $11r = 21$

(C) $5r = 14$

(D) $3r = 11$

Since the rest of the problem is routine the difficulty is probably in the application of the DISTRIBUTIVE LAW.

Basically, it is stated:

$$a (b + c) = ab + ac$$

Now when any of those letters are replaced by fractions, the rule still holds

e.g. $$8 \left(\frac{b}{2} + \frac{c}{4} \right) = \frac{8b}{2} + \frac{8c}{4}$$

$$= 4b + 2c$$

With this refresher in mind, return to page 73/2 and tackle the equation again.

You can discover for yourself whether you made the correct choice.

Take the number 11 that you chose, double it and add 5 . What do you get?

Re-read the problem and interpret the relations given into algebra. You know, this is the hardest part of algebra.

Please return to page 78/2 and try this question again.

62
1

Very good. You made the correct choice. We have to find the solution set of the equation

$$7x = 45 + 2x$$

$7x = 45 + 2x$ ⟨- $2x$ Since we would like to have the variables on one side of the equation, we subtract $2x$ from both members.

$5x = 45$ ⟨÷ 5 Now using the DIVISION PROPERTY OF EQUALITY to eliminate the 5 , we get

$x = 9$

Please go on to the next question, below.

62
2

Question 6

Apply your knowledge and find the solution set of the equation

$$9z - 24 = 3z$$

Select the letter next to the correct answer.

(A) $\{4\}$

(B) $\{8\}$

(C) $\{6\}$

(D) None of these.

$\frac{63}{1}$

We do not agree.

One of the letters does have the correct answer next to it.

Please reconsider. Review each of your steps.

Return to page $\frac{68}{2}$ and try this question again.

$\frac{63}{2}$

This problem is presented to give you one more chance to simplify parentheses in the course of solving an equation.

Now wouldn't it be smarter to change $(9 - 4)^2$ into $(5)^2$ than to attempt to multiply $(9 - 4)$ by $(9 - 4)$?

The left side of the equation is another of the type requiring a double use of the DISTRIBUTIVE LAW. That " $\frac{1}{2}$ " is merely the final multiplier.

Don't forget to collect like terms between the two uses of the [D .

Return to page $\frac{70}{2}$ and do the problem correctly.

$\frac{64}{1}$

We don't agree. Please read a problem a few times until you are thoroughly familiar with what is given and what is required. Then, let x equal the quantity that is being sought. Translate the phrases of the problem into algebraic expressions, then put these expressions into equation form. Find the solution set of this equation, and then check the answer you get with the conditions of the problem.

Return to page $\frac{82}{2}$ and try this question again.

$\frac{64}{2}$

The choice you made is incorrect.

The sum of 31 and 35 is 66 , but the difference between these two numbers is 4 and not 8 .

Return to page $\frac{74}{2}$ and try this question again.

Very good. You made the correct choice. We have to find the solution set of the equation

$$8m + 4 + 4m + 16$$

We begin by writing an equivalent equation with the variables appearing on the left side only. Thus,

$8m + 4 = 4m + 16$	$\swarrow -4m$	
$4m + 4 = 16$	$\swarrow -4$	Next, we subtract 4 from both members of the equation,
$4m = 12$	$\swarrow \div 4$	Finally, we divide by the coefficient of the variable,
$m = 3$		

Of course, we check by substituting this value in the original equation

$$8(3) + 4 \stackrel{?}{=} 4(3) + 16$$
$$24 + 4 \stackrel{?}{=} 12 + 16$$
$$28 = 28 \checkmark$$

Please go on to question 8 below.

Question 8

Apply the proper principles and find the solution set of the equation,

$$12r - 3 = 4 - 2r$$

Select the letter next to the set that contains the solution set as a subset.

(A) $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

(B) $\{2, 3, 4\}$

(C) $\left\{\frac{1}{10}, \frac{3}{10}, \frac{7}{10}\right\}$

(D) $\{6, 8, 10\}$

$\frac{66}{1}$

Very good. You made the correct choice. We have to find the solution set of the equation

$$16\left(\frac{y}{4} - \frac{1}{8}\right) = y + 40 \qquad [\ D$$

$$4y - 2 = y + 40 \qquad \ll -y$$

$$3y - 2 = 40 \qquad \ll +2$$

$$3y = 42 \qquad \ll \div 3$$

$$y = 14$$

Of course we check this:

$$16\left(\frac{14}{4} - \frac{1}{8}\right) \stackrel{?}{=} (14) + 40$$

$$16\left(\frac{28}{8} - \frac{1}{8}\right) \stackrel{?}{=} 54$$

$$16\left(\frac{27}{8}\right) \stackrel{?}{=} 54$$

$$2\ (\ 27\ ) \stackrel{?}{=} 54$$

$$54 = 54$$

Note: We don't use the DISTRIBUTIVE LAW in the check, but instead simplify the terms inside the parentheses first. In this way, we will catch any error made in applying the law.

Please go on to question 13 below.

$\frac{66}{2}$

Question 13

Apply your knowledge and find the solution set of the equation,

$$2\ [\ 2 + 3\ (\ x + 4\ )\ ]\ =\ 52$$

Select the letter next to the set that contains the solution as a sub-set.

(A) $\{8\ ,\ 10\ ,\ 12\}$

(B) $\left\{\frac{7}{5}\ ,\ \frac{9}{5}\ ,\ \frac{11}{5}\right\}$

(C) $\left\{\frac{14}{3}\ ,\ \frac{17}{3}\ ,\ \frac{19}{3}\right\}$

(D) $\{2\ ,\ 4\ ,\ 6\}$

Please note that

$$8 (2 - y) = 16 - 8y$$

The number immediately before a parenthesis, multiplies each term inside the parenthesis. This is that old familiar rule the DISTRIBUTIVE PROPERTY OF MULTIPLICATION over SUBTRACTION:

$$a (b - c) = ab - ac$$

Return to page $\frac{59}{2}$ and try this question again.

Start by letting x equal the required number. Then, double this number is $2x$. If 5 is added, you now have

$$2x + 5$$

Can you write the equation?

Return to page $\frac{78}{2}$ and try this question again.

68/1

Very good. You made the correct choice. We have to find the solution set of the equation

$$4 (x + 2) + 5 = 2 (3x - 4) + x$$

$4(x + 2) + 5 = 2(3x - 4) + x$ [D Clearing the parentheses, the above equation becomes

$4x + 8 + 5 = 6x - 8 + x$ [C ^ A Now, we combine like terms and obtain

$4x + 13 = 7x - 8$ [R

Since $7x$ is greater than $4x$ you may find it convenient to use the REFLEXIVE PROPERTY OF EQUALITY and write the above equation as,

$7x - 8 = 4x + 13$ ∝< $- 4x$ Next, we subtract $4x$ from both members of the equation,

$3x - 8 = 13$ ∝< $+ 8$

$3x = 21$ ∝< $\div 3$

$x = 7$

Please go on to question 11 below.

68/2

Question 11

Apply the usual principles and find the solution set of the equation,

$$2 (k + r) - 3 = \frac{1}{3} (12 + 9k)$$

Select the letter next to the set that contains the correct answer.

(A) $\left\{ \frac{1}{2} , \frac{1}{3} , \frac{1}{4} , \frac{1}{5} \right\}$ (C) $\left\{ \frac{3}{2} , \frac{2}{3} , \frac{4}{9} , \frac{9}{4} \right\}$

(B) $\{1 , 2 , 3 , 4\}$ (D) None of these.

$\frac{69}{1}$

Did you check your answer with the conditions of the problem?

If the number 8 is tripled and then diminished by 4 , you get

$$24 - 4 = 20$$

If 8 is doubled and increased by 12 , you get

$$16 + 12 = 28$$

Since 20 cannot equal 28 , your choice is incorrect.

Please return to page $\frac{82}{2}$ and try this question again.

$\frac{69}{2}$

Please pay close attention to the following statement.

If two numbers differ by 8 , then one number is 8 more than the other.
If x denotes the smaller number,
then $x + 8$ will denote the larger one.

Please continue.

Please return to page $\frac{74}{2}$ and try this question again.

$\frac{70}{1}$

Good. Your choice is the set that contains the correct answer as a sub-set.

$2\ [\ 2 + 3(x + 4)\ ] = 52$ [D on the innermost parentheses

$2\ [\ 2 + 3x + 12\ ] = 52$ [C ^ A

$2\ [\ 14 + 3x\ ] = 52$ [D once again

$28 + 6x = 52$ ∝ - 28

$6x = 24$ ∝ ÷ 6

$x = 4$

Of course, we check; but we don't use the [D principle in the check because if we made a mistake in the solution, we'd probably make the same mistake in the check and we would have an erroneous check.

So it looks like this:

$2\ [\ 2 + 3(4 + 4)\ ] \stackrel{?}{=} 52$ (Simplifying within the parentheses.)

$2\ [\ 2 + 3(8)\ ] \stackrel{?}{=} 52$

$2\ [\ 2 + 24\ ] \stackrel{?}{=} 52$

$2\ [\ 26\ ] \stackrel{?}{=} 52$

$52 = 52$ ✓

$\frac{70}{2}$

Question 14

Apply your knowledge and find the solution set of the equation

$$\frac{1}{2}[\ 4 + 6(2x + 1)\ ] = (9 - 4)^2 + x$$

Select the letter next to the correct answer.

(A) $\{4\}$ (C) $\{\frac{20}{3}\}$

(B) $\{2\}$ (D) None of these.

$\frac{71}{1}$

We do not agree.

One of the letters does have the correct answer next to it.

Return to page $\frac{59}{2}$ and try this question again.

$\frac{71}{2}$

To get started, denote the number of 25¢ stamps purchased by x, and since there were twenty stamps in all, there would be 20 - x left for the number of 5¢ stamps purchased. Now, consider the following question:

What is the cost in pennies of all the 25¢ stamps?

What is the cost in pennies of all the 5¢ stamps?

Yes,

$25x$ = cost in pennies of the 25¢ stamps,

$5 (20 - x)$ = cost in pennies of the 5¢ stamps.

Can you now write an equation?

Return to page $\frac{83}{2}$ and try this question again.

$\frac{72}{1}$

We do not agree.

If you took the trouble to check the choice you made, you would have discovered for yourself that 14 cannot be the answer.

If 14 is doubled and then increased by 5 , we get

$$28 + 5 = 33$$

Return to page $\frac{78}{2}$ and try this question again.

$\frac{72}{2}$

Since the length is expressed in terms of the width, you should let x denote the width of the rectangle. The length is one inch more than twice the width. Hence, the length is 1 more than $2x$ or as the expression

$$2x + 1$$

The problem then gives a relationship between the length and the width. Re-read the problem.

Can you now write an equation? Alright, now solve it.

Return to page $\frac{77}{2}$ and try this question again.

Very good. You made the correct choice. We have to find the solution set of the equation,

$2(k + 4) - 3 = \frac{1}{3}(12 + 9k)$ [D Simplifying the expressions with the parentheses

$2k + 8 - 3 = 4 + 3k$ [C^A

$2k + 5 = 4 + 3k$ ∝ - 2k

$5 = 4 + k$ ∝ - 4

$1 = k$

Please go on to question 12 below.

Question 13

Apply the appropriate principles and find the solution set of the equation,

$$16\left(\frac{y}{4} - \frac{1}{8}\right) = y + 40$$

Select the letter next to the set that contains the solution set as a subset.

(A) $\{24, 32, 40\}$

(B) $\{12, 14, 16\}$

(C) $\{2, 4, 8\}$

(D) $\{39\frac{1}{8}, 40\frac{1}{4}, 42\frac{1}{8}\}$

$\frac{74}{1}$

Very good. You made the correct choice.

Part I: The Variables:

We begin by

letting x = the required number.

This number tripled and diminished by 4

is represented by $3x - 4$

This number doubled and increased by 12

is represented by $2x + 12$

Part II: The Relationship:

The resulting numbers are equal. Therefore, we have the equation $3x - 4 = 2x + 12$

$$3x - 4 = 2x + 12 \quad \text{<- } 2x$$

$$x - 4 = 12 \quad \text{<+ } 4$$

$$x = 16$$

Part III:

We check with the English rather than the equation.

16 tripled = 48

diminished by 4 = 44

16 doubled = 32

enlarged by 12 = 44

This checks with the original problem.

Please go on to question 17 below.

$\frac{74}{2}$

Question 17

Apply your knowledge and select the letter next to the correct answer to the following problem: The sum of two numbers is 66 and their difference is 8 ; find the numbers.

(A) 31 and 35

(B) The smaller number is 8 .

(C) The larger number is greater than 35 .

(D) The smaller number is greater than 30 .

Very good. You made the correct choice.

Part I: The Variables:

Since together, they have twenty-four marbles, we can

let x = the number of marbles that Fred started with

$24 - x$ = the number of marbles that Ali started with

Now, after Ali gives Fred one marble,

$x + 1$ = the number that Fred now has

$23 - x$ = the number that Ali now has.

Part II: The Relationship:

Since Fred's new amount is twice the new amount that Ali has, we have the equation,

$$x + 1 = 2(23 - x) \quad [\text{D}$$

$$x + 1 = 46 - 2x \quad \propto +2x$$

$$3x + 1 = 46 \quad \propto -1$$

$$3x = 45 \quad \propto \div 3$$

$$x = 15$$

Thus, Fred had 15 marbles to start, and Ali had $24 - 15$ or 9 to start.

You have now finished this segment. and in your PUNCH CARD.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT.

Problems 5 , 6 , 7 , and 8 .

76/1

Very good. You made the correct choice. We have to find the solution set of the equation,

$$12 + 8(2 - y) = 6y$$

$12 + 8(2 - y) = 6y$ [D Clearing parentheses, using the DISTRIBUTIVE LAW.

$12 + 16 - 8y = 6y$ [C ^ A

$28 - 8y = 6y$ [R By the REFLEXIVE PROPERTY OF EQUALITY,

we can write $6y = 28 - 8y$ ∝< + 8y Adding 8y to both members of the equation.

we obtain $14y = 28$ ∝< ÷ 14 Dividing by 14 ,

we get $y = 2$

This value is then checked:

$12 + 8(2 - (2)) \stackrel{?}{=} 6(2)$

$12 + 8(0) \stackrel{?}{=} 12$

$12 = 12$

Please go on to question 10 below.

76/2

Question 10

Apply the proper principles and find the solution set of the equation

$$4(x + 2) + 5 = 2(3x - 4) + x$$

Select the letter next to the correct answer.

(A) $\{7\frac{1}{2}\}$ (C) $\{1\}$

(B) $\{5\}$ (D) $\{7\}$

Very good. You made the correct choice.
If the sum of two quantities is given and one of the quantities is denoted by x, then the other quantity is denoted by that sum minus x

Thus,

Part I: The Variables:

Let x = the number of 25¢ stamps purchased,

$20 - x$ = the number of 5¢ stamps purchased.

The value of the stamps expressed in pennies is $25x$ for the 25¢ stamps, and $5(20 - x)$ for the 5¢ stamps.

Part II: The Relationship:

the total value is \$2.80 .

$25x + 5(20 - x)$, and this is equal to \$2.80 .

Hence, we have the equation,

$$25x + 5(20 - x) = 280 \qquad [\text{D}$$

$$25x + 100 - 5x = 280 \qquad [\text{C} \wedge \text{A}$$

$$20x + 100 = 280 \qquad \text{<- } 100$$

$$20x = 180 \qquad \text{<: } 20$$

$$x = 9$$

$$20 - x = 11$$

Thus, the purchase consisted of nine 25¢ stamps and eleven 5¢ stamps.

Please go on to question 19 below.

Question 19

Apply your knowledge and select the letter next to the correct answer to the problem: The sum of the length and width of a rectangle is 34 inches. The length is 1 inch more that twice the width. Find the dimensions of the rectangle.

(A) The dimensions are fractions.

(B) The length is less than 21 inches.

(C) The width is greater than 12 inches.

(D) The length minus the width is 12 inches.

$\frac{78}{1}$

You picked the right choice this time.

Let's run through the technique again.

Given:	$\frac{1}{2}[4 + 6(2x + 1)]$	$=$	$(9 - 4)^2 + x$	$[C \wedge A$
	$\frac{1}{2}[4 + 6(2x + 1)]$	$=$	$5^2 + x$	$[D$
	$\frac{1}{2}[4 + 12x + 6]$	$=$	$25 + x$	$[C \wedge A$
	$\frac{1}{2}[10 + 12x]$	$=$	$25 + x$	$[D$
	$5 + 6x$	$=$	$25 + x$	$\propto -x$
	$5 + 5x$	$=$	25	$\propto -5$
	$5x$	$=$	20	$\propto \div 5$
	x	$=$	4	

Now 4 can be substituted for x on any of these lines. If the result is a true statement, then each line following is assumed to be true.

Therefore, when we check in the first line, we prove the entire problem.

Please proceed to question 15 below.

$\frac{78}{2}$

Question 15

Apply your knowledge and select the letter next to the correct answer to the following problem: When a number is doubled and then increased by 5 , it equals 17; find this number.

(A) 11

(B) Greater than 11 , but less than 14 .

(C) Less than 8 , but greater than 5 .

(D) 14

79/1

We don't agree. Always check the reasonability of your choices. If the smaller number is greater than 30 , the larger one must be greater than 38 . Hence, the sum would be greater than 68 . This contradicts the condition of the problem.

Please return to page 74/2 and try this question again.

79/2

We don't agree.

To get started, consider.

Part I: The Variables:

Let x = the number of marbles that Fred had and let

$24 - x$ = the number of marbles that Ali had.

Keep in mind that an exchange of one marble means that one will have one more than before, and the other will have one less than before.

Part II: The Relationship:

Another point to keep in mind is the relationship between the new amounts as stated in the problem:

"Fred will have twice as many as Ali."

Please return to page 85/2 and go on to the next question.

80
1

VOLUME 4 SEGMENT 3 BEGINS HERE:

Obtain a Punch Card from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48 and 50	1	8	(Sequence Number)
	54 and 56	0	4	(Type of Punch Card)
	60 and 62	0	4	(Volume Number)
	66 and 68	0	3	(Segment Number)

Your reading assignment for this Segment is pages 111 - 115.

SUPPLEMENTARY NOTES:

In your work in the previous segment, you might have noticed that whenever you applied the ADDITION or SUBTRACTION PROPERTY of EQUALITY in solving an equation, on one side of the equation you employed the additive inverse and arrived at a zero result; while on the other side of the equals sign, the operation was always possible since a smaller number was being subtracted from a larger number. Did you ever question what would happen if you tried to subtract a larger number from a smaller one?

In this segment, we will explore this situation and you will be introduced to the set of NEGATIVE NUMBERS. You will realize that every number is a DIRECTED NUMBER, and further, that two numbers are either equal or one is larger than the other.

You will now be asked a series of questions to draw your attention to the more important points.

80
2

Which do you recognize as the proper meaning of $^{+}4$ in reference to the NUMBER LINE ?

A. add 4

B. 4 to the right of zero

C. a distance of 4

D. 4 more than

A point on the NUMBER LINE has both a distance and a direction relative to the zero point assigned to it.

The coordinate $^{-}1$ indicates that the point is to be to the left of zero. However, point Q is to the right of zero. It is, therefore, not the correct point, even though its distance from zero is 1 unit.

Return to page 108/2 and try this question again.

If you make a NUMBER LINE and mark off points with coordinates 0 , $^{+}1$, $^{+}2$, etc., you can do the counting right on the line. If you do this, you will find that this choice is not correct.

Return to page 107/2 and try the question again.

$\frac{82}{1}$

Very good. You made the correct choice.

We begin by letting x = the required number.

This number doubled, becomes $2x$ and when it is increased by 5, it becomes

$$(2x + 5)$$

Now, this last expression is equal to 17. Hence, the answer to this problem is the solution set of the equation

$$2x + 5 = 17$$

Subtracting 5 from both members of the equation, we get

$$2x = 12 \quad \text{and} \quad x = 6$$

Please go on to question 16 below.

$\frac{82}{2}$

Question 16

Apply the proper principles and select the letter next to the correct answer to the following problem:

"When a certain number is tripled and then diminished by 4, it equals this number doubled and increased by 12."

Find this number.

(A) The number is a fraction less than 8.

(B) 8

(C) Greater than 15.

(D) Less than 15, but greater than 8.

Very good. You made the correct choice.

When two numbers differ by 8, the larger number is 8 more than the smaller one.

Part I: The Variables:

Let x = the smaller number.

$x + 8$ = the larger number.

Part II: The Relationship:

The sum = 66

The sum of the two numbers is

$(x) + (x + 8) = 2x + 8$

and this is equal to 66.

Hence, we have the equation

$$2x + 8 = 66 \quad - 8$$
$$2x = 58 \quad \div 2$$
$$x = 29$$

Thus, the smaller number is 29 and the larger one is 37.

Please go on to question 18 below.

Question 18

Apply the technique of setting up and solving an equation and select the letter next to the correct answer to the following problem:

A purchase of 20 stamps, some costing 5¢ each, and some costing 25¢ each, cost $2.80; how many of each kind are there?

(A) There were 8 more 25¢ stamps

(B) There were 4 more 25¢ stamps

(C) There were 6 more 25¢ stamps

(D) None of these.

$\frac{84}{1}$

When the minus sign is written in the usual position in front of a number, it indicates "subtract" or "less than."

Thus -3 means "subtract 3" or "3 less than." In any case, it does not indicate direction.

Return to page $\frac{91}{2}$ and try this question again.

$\frac{84}{2}$

If you count spaces carefully, you will find that W is five spaces from zero. However, it is not to the right of the zero point, which is indicated by the coordinate $^{+}5$. Therefore, this choice is not correct.

Return to page $\frac{101}{2}$ and try this question again.

Very good. You made the correct choice.
Let's review the technique together.

Part I: The Variables:

Let x = the width of the rectangle,

$2x + 1$ = the length of the rectangle, since the length is one more than twice the width.

Part II: The Relationship:

The problem states:

The sum of the length and width is equal to 34 . We, therefore, have the equation,

length	+	width	=	34	
$(2x + 1)$	+	(x)	=	34	[C^A
$3x$	+	1	=	34	$\propto -1$
$3x$			=	33	$\propto \div 3$
x			=	11	
and $2x$	+	1	=	23	

Thus, the dimensions of the rectangle are 11 for the width, 23 for the length. This choice stated the length minus the width is 12 inches, and 23 - 11 does equal 12 .

Please go on to question 20 below.

Question 20

Apply the principles you've learned and select the correct answer to the following problem: Fred and Ali together have twenty-four marbles. If Ali loses one marble to Fred, Fred will have twice as many as Ali will then have. How many marbles did each one have to begin with?

(A) Fred had 9 more marbles than Ali.
(B) Ali had 5 marbles less than Fred.
(C) Fred had 14 marbles.
(D) Ali had 9 marbles.

$\frac{86}{1}$

You should watch the position of the plus sign with respect to its height above the line. We use the sign + to represent addition, but you have found another meaning for the symbol $^{+}$ (a raised plus sign) in front of a number. Thus " +2 " means " add 2 , " while " $^{+}2$ " has a different meaning. It is the other meaning that you are looking for here.

Return to page $\frac{80}{2}$ and try question 1 again.

$\frac{86}{2}$

Since point R represents a depth of 500 feet, its coordinate should be $^{-}500$. But you were asked for the magnitude, which means the distance from zero, without direction being attached. Since this has a direction, it is not correct.

Return to page $\frac{110}{2}$ and try this question again.

A point on the NUMBER LINE has both a distance and a direction assigned to it, relative to the zero point.

The coordinate $^{-}1$ indicates that the point is 1 unit to the left of zero. It is true that R is to the left of zero, but is it 1 unit? The number at point R is $^{-}4$, which indicates that it is 4 units to the left.

Return to page 108/2 and try the question again.

The opposite of a number N is the number which is located on the opposite side of zero but has the same magnitude as N. It will, therefore, have the opposite sign. However, it should have the same magnitude. Your choice does have the correct sign; but you chose the reciprocal, not the opposite. The reciprocal of ^{+}a is $^{+}\frac{1}{a}$.

The opposite of ^{+}a is ^{-}a.

Return to page 109/2 and try question 9 again.

88/1

Since F represents a gain of \$5, it should be located on a NUMBER LINE with a coordinate of $^{+}5$. G should have a coordinate of $^{-}3$, since it represents a loss of \$3. Then, starting at G , it is necessary to pass zero on the way to F . But this choice says that you count two units to the left. If you locate the points on a NUMBER LINE, you will find that both the distance and the direction are incorrect.

Return to page 94/2 and try the question again.

88/2

Did you notice that since the scale showed $^{+}1.0$ as the second mark, each mark must equal a 0.5 step?

You were asked to locate first the point whose coordinate is $^{+}1.5$; but your choice lists F as the first point. F does not have the coordinate $^{+}1.5$; therefore, this choice is not correct. Did you locate the three points desired and then find their order on the NUMBER LINE? That was not what was asked for. You were asked for the points in the same order as the coordinates listed in the question.

Return to page 102/2 and try question 8 again.

$\frac{89}{1}$

The coordinate $^{+}5$ indicates that the point is to the right of the zero point. But point X is to the left, and if you check carefully, you will find that it is not five units from zero. Although you start at zero, you don't begin to count until you reach the first mark.

Return to page $\frac{101}{2}$ and try question 4 again.

$\frac{89}{2}$

The point whose coordinate is $^{-}5$ is five units to the left of zero. We shall call it point P for easy reference. Since the point $^{+}2$ is to the right of the zero point, it cannot be to the left of P. Then this choice is not correct. In problems of this sort it usually is easier to interpret by sketching a NUMBER LINE, entering the facts, and then making a decision.

Return to page $\frac{105}{2}$ and try question 11 again.

$\frac{90}{1}$

If you wish to indicate that the man borrows \$50, you should use the minus sign getting $^{-}50$. Since this choice disagrees with the number you chose, it is not correct.

Return to page $\frac{114}{2}$ and try question 10 again.

$\frac{90}{2}$

As the NUMBER LINE will show you, if you count 10 units to the left, and then 3 units to the right, you will finish to the left of zero. Then the point you reach will not have a positive coordinate. Therefore, this choice is not correct. Making the NUMBER LINE and marking points on it is necessary to the understanding of these questions.

Return to page $\frac{111}{2}$ and try question 12 again.

The fact that the plus sign is written in a raised position (about the same height as an exponent) indicates that it is to be considered as a sign of direction, rather than operation. $^{+}4$ means a distance of four units to the right of zero on the NUMBER LINE. Therefore, this choice is correct.

Proceed to question 2 below.

Question 2

Which number do you recognize as one representing three units to the left of zero?

(A) $^{-}3$

(B) -3

(C) 3-

(D) $\overleftarrow{3}$

92/1

Since point R represents a depth of 500 feet, its coordinate should be $^{-}500$. But you were asked for the magnitude. which means the distance from zero, without direction being attached. Not only does this choice have a direction attached, but it is positive. That would mean height above sea level, rather than depth below it. A magnitude cannot have a sign attached, therefore, this choice is not correct.

Return to page 110/2 and try question 7 again.

92/2

The opposite of a number N is the number which is located on the opposite side of zero, but has the same magnitude as N. Since this choice is positive 2 , it is the same as the given number, not its opposite. The opposite of ^{+}a is ^{-}a.

Return to page 109/2 and try this question again.

You must recognize that a point on the NUMBER LINE has both a distance from zero and a direction from zero assigned to it.

The coordinate $^{-}1$ indicates that the point is one unit to the left of zero. However, S is to the right of zero. In addition, it is more than four units to the right, as you should observe. Therefore, this choice is not correct.

Return to page 108/2 and try question 3 again.

Did you notice that $^{+}1$ was located at the second mark on the NUMBER LINE from zero? Therefore, you should conclude that each division represents a space equivalent to .5 units.

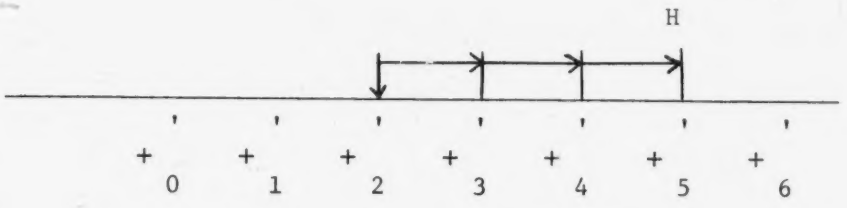
The second point you were asked to locate was the point with coordinate $^{-}.5$. Your choice lists K as the second point, that is, the one whose coordinate is $^{-}.5$. But point K is to the right of zero, and therefore, it has a positive coordinate. Then this choice is not correct, even though one of the points is correct. Can you find which other point is also incorrect.

Return to page 102/2 and try the question again.

$\frac{94}{1}$

If you can make a NUMBER LINE, and on it you mark off zero and $^{+}1$, $^{+}2$, etc., you can do the counting right on the line.

Using the line below, you will find that H does have the coordinate $^{+}5$. Then this choice is correct.



Please proceed to question 6 below.

$\frac{94}{2}$

Question 6

On a NUMBER LINE F is a point representing a gain of $5 , and, at the same time, G represents a loss of $3 .

Starting at G , apply your knowledge to find the distance and direction to F .

(A) - 2

(B) 8

(C) - 8

(D) + 8

The coordinate $^{+}5$ indicates that the point is to the right of zero. Point Y is indeed to the right, but is it five units from zero? Although you start at zero, you don't count " 1 " until you move to the first mark.

Return to page 101/2 and try question 4 again.

The point P whose coordinate is $^{-}5$ is located 5 units to the left of zero. Then 0 is to the right of P , not to its left. Therefore, this choice is not correct.

It is usually easier to interpret problems of this sort by sketching a NUMBER LINE, entering the facts, and then making a decision.

Return to page 105/2 and try the question again.

96/1

A distance of 4 is represented by the number 4 , but what has the plus sign to do with it? This choice is not correct, since the plus sign has to have a meaning. The raised sign indicates which direction from the zero point on the NUMBER LINE is intended, and the number itself indicates the distance.

Return to page 80/2 and try question 1 again.

96/2

As the NUMBER LINE will show you, if you count ten units to the left, and then three units to the right, you will finish to the left of zero. If you make the NUMBER LINE, and count carefully, you will find that the magnitude of this answer is not correct, although its sign is correct.

Return to page 111/2 and try the question again.

97/1

The correct coordinate of R is -500, since it represents a depth of 500 feet. Its magnitude is its distance from zero, regardless of direction. In addition, the magnitude is a number. But this choice offers 500 feet, which is not a pure number, but a denominate quantity, that is, a quantity of specific units. It is important for us to understand that we use numbers, not denominate quantities, in mathematics. Of course, when we interpret results, we usually attach some type of unit to the number. But this choice is not correct.

Return to page 110/2 and try the question again.

97/2

The opposite of a number N is the number on the opposite side of zero, but which has the same magnitude as N. Since this choice does not have a magnitude of 2 (the magnitude of the given number, positive 2) it cannot be the opposite. You actually chose the reciprocal of $+2$ which is $+\frac{1}{2}$

Return to page 109/2 and try the question again.

98
1

In mathematics, symbols are used in certain standard ways. There is no accepted meaning to a minus sign following a number; the usual meaning of 8 - 2 is "subtract 2 from 8," or "2 less than 8," or "8 diminished by 2." Thus we read the minus sign as being in front of the 2, not following the 8. Since this choice has a minus sign with no number after it, it has no meaning.

Return to page 91/2 and try this question again.

98
2

In dealing with signed numbers, we must learn that larger means further to the right on the NUMBER LINE. Then, if the numbers are in order of size, from smallest to largest, the points which have these coordinates will be on a NUMBER LINE, in order, from left to right.

If you locate these numbers on a line, you will find that the points are not in the proper order.

Return to page 106/2 and try the question again.

Since F represents a gain of \$5 , it should be given a coordinate of $^{+}5$ on a NUMBER LINE.

Since G represents a loss of \$3 , it should be given a coordinate of $^{-}3$ on a NUMBER LINE.

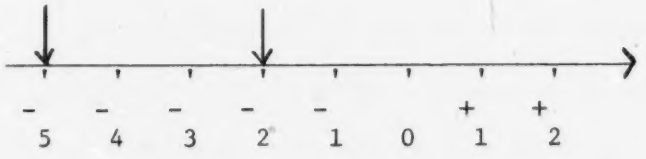
Then you do find that it is 8 units from G to F , but this choice did not give a sign to indicate the direction.

While it is possible to argue that this choice should be considered correct, there is a better choice.

Please return to page $\frac{94}{2}$ and try question 6 again.

Both points, $^{-}5$, and $^{-}2$, are located to the left of zero.

If you locate them on a NUMBER LINE, you will find that $^{-}2$ is not to the left of $^{-}5$.



Please return to page $\frac{105}{2}$ and try this question again.

100/1

You should note the position of the plus sign with respect to the number. We use the sign + to represent more than, but your text has given you another meaning for the symbol $^{+}$ when written in a raised position in front of a number. Thus +2 means " 2 more than, " but $^{+}2$ has a different meaning. It is this other meaning that you must find.

Return to page 80/2 and try the question again.

100/2

If we just considered only the magnitude of the number and not its sign, then -7 (with the magnitude 7) would be larger than 3 , or 3 would be less than 7 . However, the way to consider this question is to refer to the NUMBER LINE.

The number $^{-}7$ is located to the left of zero on the NUMBER LINE. Since the number 3 is located to the right of zero, it is to the right of $^{-}7$. But if one point is to the right of another, its coordinate is larger than the coordinate of the other. Therefore, 3 is larger than $^{-}7$, not less.

Return to page 118/2 and try question 14 again.

A point on the NUMBER LINE has both a distance and a direction assigned to it.

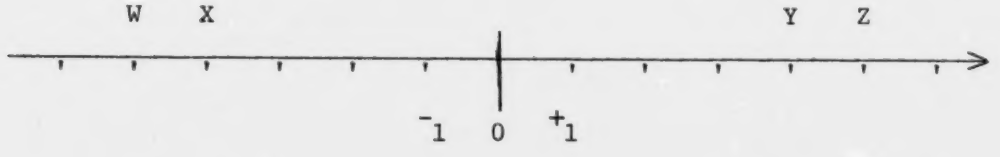
It is absolutely necessary that the scale on the NUMBER LINE be uniformly divided, and that the point representing zero be indicated. Every point need not be labeled, but the units should be so obvious that the observer can determine the coordinate of any desired point.

The coordinate $^{-}1$ indicates 1 unit to the left of zero. Since point P is between 0 and the point marked $^{-}2$, it indicates that it is one unit from zero. Since it is to the left of zero, it is the point we were to find.

Please proceed to question 4 below.

Question 4

Which point do you recognize as having the coordinate +5 on the NUMBER LINE below?



(A) W

(B) X

(C) Y

(D) Z

102
1

Since point R represents a depth of 500 feet, its coordinate should be -500. That means that its distance from zero is 500 units, and it is to the left of zero. But you were asked for the magnitude, which means the distance alone, without direction.

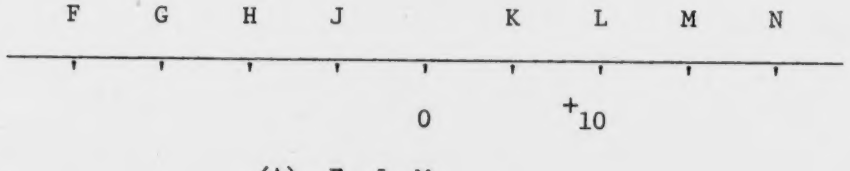
Then the magnitude is 500, and this choice is correct.

Please proceed to question 8 below.

102
1

Question 8

On the NUMBER LINE below, choose the points whose coordinates are, in order, positive 15, negative 5, and negative 20.



(A) F, J, M

(B) M, K, N

(C) G, K, F

(D) M, J, F

The arrow above the numeral 3 certainly suggests going to the left three units. However, this is not the conventional mathematical way of showing it. What was the method of indicating this which your text described? Remember that all language is based on convention, and is used for communication. If you wanted to use the arrow symbol, and enough people were willing to go along with you, it could be used. However, we are trying to learn ideas in mathematics, and we will use conventional symbols and language in our work. It is important for you to learn them.

Return to page 91/2 and try question 2 again.

If you locate on a NUMBER LINE, point P whose coordinate is $^{-}3$ and the point whose coordinate is $^{-}4$, you will find that $^{-}4$ is to the left of point P. Then this choice is not correct.

Return to page 129/2 and try the question again.

104/1

Since F represents a gain of \$5, it should be given a coordinate of $^{+}5$ on a NUMBER LINE. Since G represents a loss of \$3, its coordinate should be $^{-}3$. If you locate these two points on a NUMBER LINE, and count from G to F, you will discover that you are not counting to the left. But the choice $^{-}8$ indicates that the direction is to the left. Then this choice is not correct.

Return to page 94/2 and try this question again.

104/2

This choice indicates that points P, Q and R are located on the NUMBER LINE in that order, reading from left to right. If you locate them on a NUMBER LINE, you will find that this is not correct.

Please return to page 124/2 and try the question again.

If you wish to indicate that a man borrows \$50 , you should use the minus sign, getting $^{-}50$. To represent the earning of \$80 , we use a positive sign, getting $^{+}80$. Then this choice is correct.

Please proceed to question 11 below.

Question 11

Choose which of the following numbers would be located on a NUMBER LINE to the left of the point representing negative 5 .

(A) $^{+}2$

(B) $^{-}10$

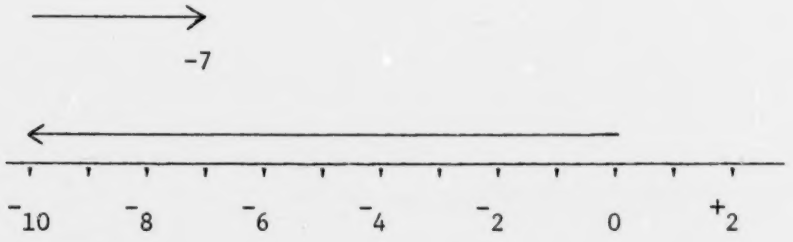
(C) 0

(D) $^{-}2$

106
1

As the NUMBER LINE will show you, if you count 10 units to the left and then 3 units to the right; you will finish to the left of zero.

Counting carefully, you find that the finish point is 7 units from zero. Then the coordinate of the finish point is $^{-}7$, and this choice is correct.



Please proceed to question 13 below.

106
2

Question 13

Of the three signed numbers $^{-}13$, $^{+}2$, $^{-}4$, choose which is the order of size from smallest to largest.

(A) $^{-}13$, $^{+}2$, $^{-}4$

(B) $^{+}2$, $^{-}4$, $^{-}13$

(C) $^{-}4$, $^{+}2$, $^{-}13$

(D) $^{-}13$, $^{-}4$, $^{+}2$

Note that once zero and $^{+}1$ are located, the remainder of the scale is accepted as being consistent with that basic unit.

The coordinate $^{+}5$ indicates that the point is to the right of the zero point and is five units from it. Point Z is indeed to the right of zero. Also, it is five units from zero. Then this choice is correct.

Proceed to question 5 below.

Question 5

Starting from the point whose coordinate is $^{+}2$, count three spaces to the right and label the point H. Apply your knowledge to find the coordinate of point H.

(A) $^{+}5$

(B) $^{+}4$

(C) $^{+}3$

(D) $^{+}6$

108
1

Since the minus sign is written in a raised position in front of the number, it indicates the direction of the number.

Our convention is that to the right of zero on the NUMBER LINE is $^{+}$ and to the left of zero on the NUMBER LINE is $^{-}$.

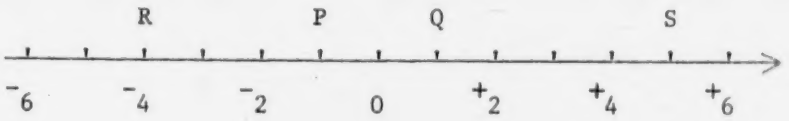
Therefore, this choice is correct.

Please proceed to question 3 below.

108
2

Question 3

Which point do you recognize as having the coordinate $^{-}1$ on the NUMBER LINE below?



(A) P

(B) Q

(C) R

(D) S

You were asked to find three points with given coordinates in the order listed in the question. Then the first point with coordinate

$$^{+}15 \quad \text{is} \quad M$$

The second point whose coordinate is

$$^{-}5 \quad \text{is} \quad J$$

The third point with coordinate

$$^{-}20 \quad \text{is} \quad F$$

Therefore, this choice is correct.

Please proceed to question 9 below.

Question 9

Choose the opposite of the number positive 2 .

(A) $^{-}\frac{1}{2}$

(B) $^{+}2$

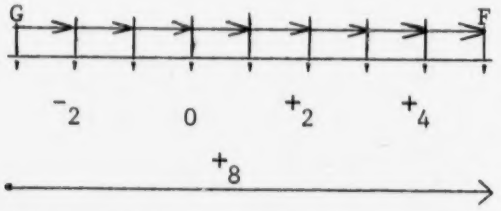
(C) $^{-}2$

(D) $^{+}\frac{1}{2}$

Since F represents a gain of $5 it should be given a coordinate of $^{+}5$ on a NUMBER LINE. Since G represents a loss of $3 its coordinate should be $^{-}3$. Locating these two points on the NUMBER LINE below, we count <u>from</u> G <u>to</u> F according to the instructions.

We find that we count 8 units to the right. But that is exactly what is meant by the signed number $^{+}8$. Then this choice is correct.

Note that choice B, which is 8 is not considered to be correct because there is no sign written in front of it. It is true that 8 is interpreted the same as $^{+}8$, but the sign ought to be written when direction is desired.



Please proceed to question 7 below.

<u>Question 7</u>

If point R on a NUMBER LINE represents an ocean depth of 500 feet, choose the magnitude of the number corresponding to point R.

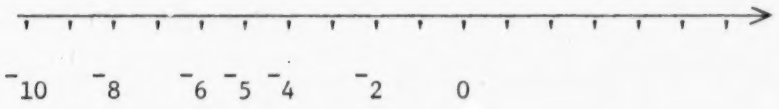
(A) 500 (C) $^{+}500$

(B) $^{-}500$ (D) −500 feet

$\frac{111}{1}$

The Point P whose coordinate is $^{-}5$ is located 5 units to the left of zero. The point offered in this choice, $^{-}10$, is 10 units to the left of zero.

If you make a NUMBER LINE and mark these points on it, you will find that $^{-}10$ is to the left of $^{-}5$.



Therefore, this choice is correct.

$\frac{111}{2}$

Question 12

Start at zero on a NUMBER LINE; go 10 units in a negative direction, and then from that point, 3 units in a positive direction. Apply your knowledge to find the coordinate of the point at which you finished.

(A) $^{+}7$

(B) $^{+}13$

(C) $^{-}7$

(D) $^{-}13$

112/1

If you just considered the absolute magnitude of $^{-}7$ and $^{-}3$, then it would be true that $^{-}3$ is less than $^{-}7$. But this is not the intention of the question. You must consider BOTH the absolute magnitude and the direction sign. The relative positions on the NUMBER LINE determine which is smaller.

If you locate the numbers $^{-}7$ and $^{-}3$ on a NUMBER LINE, you will discover that $^{-}3$ is to the right of $^{-}7$. Therefore the number $^{-}3$ is larger than $^{-}7$. But you were looking for a number less than $^{-}7$. Then this choice is not correct.

Return to page 118/2 and try the question again.

112/2

This choice indicates that n can have any value <u>between</u> $^{-}2$ and 1. But the graph shows, by means of the solid circles at the end points, that the end points are part of the domain. Then this choice is not correct.

Return to page 130/2 and try this question again.

In dealing with signed numbers, we must learn that larger means further to the right on the NUMBER LINE. Then, if the numbers are in order of size, from smallest to largest, the points which have these coordinates will be on a NUMBER LINE, in order, from left to right.

If you locate these numbers on a line, you will find that the points are in order. But they are not in order, from left to right; rather, they are in reverse order, from largest to smallest.

Return to page 106/2 and try the question again.

This choice indicates that the points Q , P and R are located on a NUMBER LINE, in that order, reading from left to right. If you locate them on a NUMBER LINE, you will find that this is not correct.

Return to page 124/2 and try this question again.

114
1

The opposite of a number N is the number which is located on the opposite side of zero, but has the same magnitude as N . That is, the opposite of ^{+}a is ^{-}a .

Since this choice is negative 2 , it is on the opposite side of zero from positive 2 , and its magnitude is also 2 . Then this choice is correct.

Proceed to question 10 below.

114
2

Question 10

If a man borrows \$50 and then earns \$80 , choose the signed numbers he might use to represent these facts.

(A) $^{+}50$, $^{+}80$

(B) $^{+}50$, $^{-}80$

(C) 50 , $^{+}80$

(D) $^{-}50$, $^{+}80$

The point whose coordinate is $^{-}3$ and the point P which represents negative 3 are the same point. Therefore, this cannot be the correct choice.

Please return to page 129/2 and try the question again.

Pretty good. You almost did it. Your answer is correct except for one consideration.

According to the graph of this choice, the number $^{+}4$ is not included in the solution set. The graph shows this by using the open circle at $^{+}4$. If we substitute $^{+}4$ for t in the open sentence, we get

$$2(\,^{+}4\,) + 3$$

which has the same value as

$$2(\ 4\ ) + 3$$

This equals $8 + 3 = 11$

Then this value is in the solution set. Since one of the values in the solution set is not in the choice you have selected, the choice is not correct.

Please return to page 125/2 and try this question again.

116/1

This open sentence indicates that n can have any value from $^{-}2$ to 1, including $^{-}2$, but not including 1. However, the graph, by means of the solid circles at the end points, shows that both end points are included in the domain. Then this choice is not correct.

Return to page 130/2 and try this question again.

116/2

According to this choice, his mark could be any number larger than 80, but not the number 80. Since the problem states that he did receive 80, this choice is not correct. In addition, we might note that the value 500 would fit this choice, but how could Harry get a mark of 500%?

Return to page 133/2 and try this question again.

$\frac{117}{1}$

This choice indicates that the points R , Q and P are located on a NUMBER LINE, in that order, reading from left to right. If you locate them on a NUMBER LINE, you will find that this is not correct.

Return to page $\frac{124}{2}$ and try this question again.

$\frac{117}{2}$

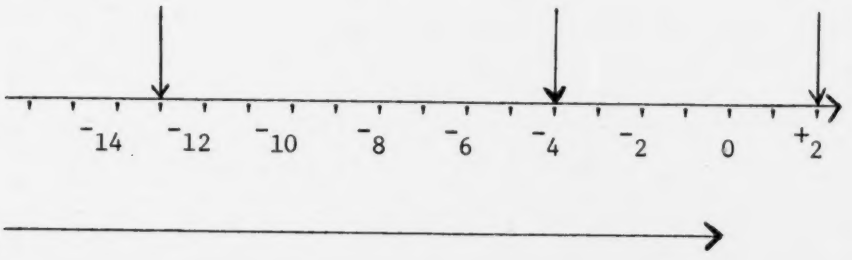
This choice says that the temperature is always between $^{-}12$ and $^{+}110$, but never reaches either value. But the problem states that the temperature does go as low as 12° below zero. Then this choice is not correct.

Return to page $\frac{137}{2}$ and try this question again.

118
1

In dealing with signed numbers, we must learn that larger means further to the right on the NUMBER LINE. Then, if the numbers are in order of size from smallest to largest, the points which have these coordinates will be on a NUMBER LINE, in order, from left to right.

If you locate these numbers on a line, you will find that the points are in the desired order.



Then this choice is correct.

Please proceed to question 14 below.

118
2

Question 14

Choose which number is less than the number $^{-}7$.

(A) 3

(B) $^{-}3$

(C) $^{-}8$

(D) 0

A positive number is represented by a point on the NUMBER LINE to the right of the point labeled 0 . Adding a positive number means moving to the right of point 0 even farther.

Since you were told to add two positive numbers, this situation would result no matter how small they were. You couldn't possibly end up to the left of zero where the negative numbers are.

Please return to page 135/1 and try this question again.

Did you try a sample case?

Consider these sums:

$$(\ ^{+}8\ ) + 0 = {}^{+}8$$

$$(\ ^{-}7\ ) + 0 = {}^{-}7$$

Did you change your mind?

Adding zero doesn't change the value of the original number. Therefore, the result is not smaller than the original number.

Please return to page 143/2 and try this question again.

$\frac{120}{1}$

This choice indicates that n can have any value between $^{-}2$ and 1, including 1, but not including $^{-}2$.

The graph shows, by means of the solid circles at the end points, that both end points are included in the domain.

Then this choice is not correct.

Please return to page $\frac{130}{2}$ and try this question again.

$\frac{120}{2}$

By using the open circle at $^{+}4$, the graph indicates that the solution set is all numbers larger than $^{+}4$. If we substitute $^{+}4$ for t, we get

$$2(\,^{+}4\,) + 3$$

which has the same value as

$$2(\,4\,) + 3$$

This equals $8 + 3 = 11$

Then $^{+}4$ is in the solution set. Since this choice said it was not in the set, the choice is not correct.

Now did you consider those values larger than 4 which were included in your choice?

Please return to page $\frac{125}{2}$ and try question 18 again.

121/1

If you locate point P , whose coordinate is $^{-}3$, on a NUMBER LINE, and also locate the point whose coordinate is $^{-}5$, you will find that $^{-}5$ is not to the right of point P .

Return to page 129/2 and try the question again.

121/2

This choice states that all marks which Harry received were less than 80 . But the problem said that 80 was the lowest mark. Therefore this choice is not correct.

Return to page 133/2 and try this question again.

122/1

This choice says that the temperature is always between $^{-}12$ and $^{+}110$ and may take either value. But the problem said that the temperature is never as high as 110° above zero.

Therefore, this choice is not correct.

Please return to page 137/2 and try this question again.

122/2

In mathematics, to disprove a statement we need only to cite one exception.

Consider the sum of

$$(\,^{+}4\,) + (\,^{-}3\,)$$

Clearly, the sum of a positive and a negative number is not <u>always</u> a negative number.

Please return to page 126/2 and try this question again.

$\frac{123}{1}$

If you locate the numbers $^{-}7$ and 0 on a NUMBER line, you find that $^{-}7$ is to the left of 0 .

Then 0 is to the right of $^{-}7$, and therefore it is larger than $^{-}7$.

But you were asked for a number less than $^{-}7$.

Therefore, this choice is not correct.

Please return to page $\frac{118}{2}$ and try this question again.

$\frac{123}{2}$

Did you draw a NUMBER LINE, or did you try to answer this question mentally?

Please draw a NUMBER LINE and follow each instruction carefully.

Please return to page $\frac{149}{2}$ and try this question again.

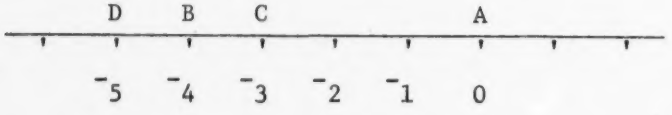
124
1

The point whose coordinate is negative 3 is located to the left of zero.

Then 0 is to the right of $^{-}3$.

Since that is what you were to find, this choice is correct.

If we examine all the choices offered by locating them on the NUMBER LINE below, we find that only this choice is correct.



Please proceed to question 16 below.

124
2

Question 16

If P represents $^{-}3$, Q represents $^{+}2$, and R represents 0 , choose the correct statement about their order.

(A) P < Q < R

(B) P < R < Q

(C) Q < P < R

(D) R < Q < P

This open sentence says that n can have any value from $^{-}2$ to 1 inclusive. The graph, by means of the solid circles at the end points, shows that both end points are included in the domain.

Therefore, this choice is correct.

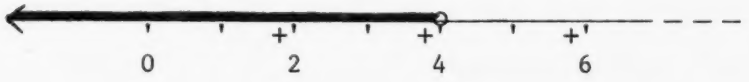
Please proceed to question 18 below.

Question 18

Apply the principles of substitution to find the graph of the solution set of the open sentence

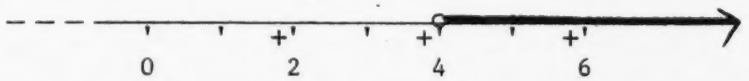
$$2t + 3 \leq 11$$

(A)



(B)

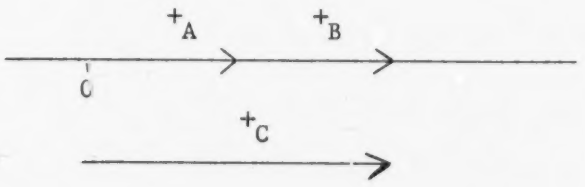
(C)



(D)

126
1

Very good. You made the correct choice. A positive number is a point on the NUMBER LINE to the right of the point labeled 0 .
The addition of a positive number means moving farther to the right as many units as the number. Hence, the addition of two positive numbers results in a point on the NUMBER LINE to the right of 0 , and is, therefore, always positive.



$(^{+}A) + (^{+}B) = (^{+}C)$

Please go on to question 2 below.

126
2

Question 2

Apply your knowledge of addition on the NUMBER LINE and select the letter which correctly answers this question: What kind of a number is the sum of a positive and negative number?

(A) A negative number

(B) A positive number

(C) Zero

(D) Sometimes positive, sometimes negative, and sometimes zero.

This choice says that the temperature is always between $^{-}12$ and $^{+}110$ and may be $^{+}110$, but may not be $^{-}12$. But the problem stated that the temperature is never as high as $110°$ above zero, and does reach a low of $12°$ below zero.

Therefore, this choice is not correct.

Return to page 137/2 and try this question again.

This choice says that all Harry's marks were between 80 and 100, with 80 as a possible mark, but not 100. What information do we have that he didn't have at least one perfect examination mark of 100?

This choice is not correct.

Return to page 133/2 and try this question again.

$\frac{128}{1}$

We don't agree.

Consider, for example, the sum of

$$(\ ^{+}15\ ) + (\ ^{-}5\ ) = {}^{+}10$$

While it is true that the result $^{+}10$ is less than $^{+}15$, the sum is not less that $^{-}5$.

Return to page $\frac{143}{2}$ and try this question again.

$\frac{128}{2}$

You have heard the expression that a picture is worth a thousand words. Start by drawing a NUMBER LINE, and follow each of the given instructions.

Don't try to do this question in your head.

Return to page $\frac{140}{2}$ and try this question again.

The simplest way to consider questions like this is to place the numbers on a NUMBER LINE. As the numbers range from left through zero to right they go from smaller to larger.

If you locate the numbers $^{-}7$ and $^{-}8$ on a NUMBER LINE, you will discover that $^{-}8$ is to the left of $^{-}7$. Therefore, the number $^{-}8$ is less than the number $^{-}7$. But that is what you were looking for.

Therefore, this choice is correct.

Proceed to question 15 below.

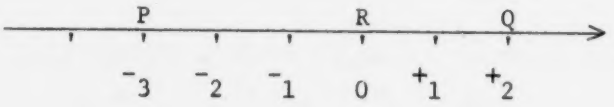
Question 15

If point P represents negative 3 on a NUMBER LINE, choose the number which indicates a point to the right of P .

(A) 0 (A) $^{-}4$ (C) $^{-}3$ (D) $^{-}5$

130
1

This choice indicates that the points P , R , and Q are located on the NUMBER LINE in that order, reading from left to right. As the diagram shows, that is correct.

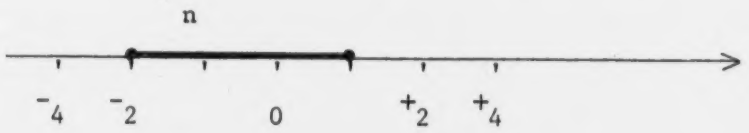


Please proceed to question 17 below.

130
2

Question 17

Apply the principles of NUMBER LINE and inequality notation to find the open sentence which has the following graph as its domain:



(A) $-2 < n < 1$

(B) $-2 \leq n < 1$

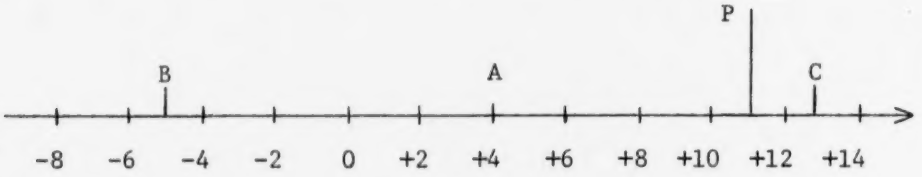
(C) $-2 \leq n \leq 1$

(D) $-2 < n \leq 1$

Very good. You made the correct choice.

We begin by drawing a NUMBER LINE. Note: each unit = 2

We start at $^{+}11$, which we'll call point P .



Now, seven units backward brings us to point A; nine units farther backward brings us to point B , ($^{-}5$) ; and eighteen units forward, brings us to point C. Hence, $^{+}13$ is the correct answer.

Please go on to question 7 below.

Question 7

Apply your knowledge and select the letter which designates the point on the NUMBER LINE you will arrive at after you follow these directions: You are $^{+}8$ facing east. Take a trip of x miles west, and then take a trip of five miles east.

(A) $^{+}x + (^{-}5)$

(B) $3x$

(C) $^{+}5 + (^{-}x)$

(D) $^{+}13 + (^{-}x)$

132
1

The graph indicates by means of the solid circle at $^{+}4$, that the solution set is all values from $^{+}4$ up.

If we substitute $^{+}4$ for t , we get $2(^{+}4) + 3$ which has the same value as

$$2\ (4)\ +\ 3$$

This is equal to

$$8\ +\ 3\ =\ 11$$

Then $^{+}4$ is a member of the solution set.

Now, suppose t has a larger value than $^{+}4$. Twice t is then larger than 8 , and adding 3 gives a result which is larger than 11 . But the open sentence called for a total less than or equal to 11 !

Therefore, since numbers larger than $^{+}4$ are not in the solution set, this choice is not correct.

Your answer must be completely correct, not partially correct!

Please return to page 125
2 and try this question again.

132
2

In mathematics, to disprove a statement, we need only one exception.

Consider the sum of

$$(^{+}5) + (^{-}7)$$

Thus, the sum of a positive and negative number is not <u>always</u> a positive number.

Please return to page 126
2 and try this question again.

This choice indicates that the temperature is always between $^{-}12$ and $^{+}110$, that it may be $^{-}12$, but not $^{+}110$. Since the problem states that the temperature may be as low as $12°$ below zero and is never as high as $110°$ above zero, this choice is correct.

Proceed to question 20 below.

Question 20

The lowest mark that Harry has received on an examination has been 80% . Apply your knowledge to find which choice describes the domain of possible marks, M , that Harry might have received.

(A) $M > 80$

(B) $M < 80$

(C) $80 \leq M < 100$

(D) $80 \leq M \leq 100$

VOLUME 4 SEGMENT 4 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

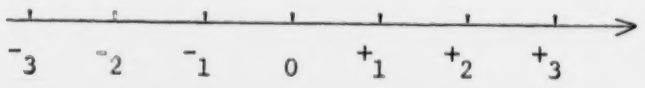
COLUMNS	48	and	50	1	9	(Sequence Number)
	54	and	56	0	4	(Type of Punch Card)
	60	and	62	0	4	(Volume Number)
	66	and	68	0	4	(Segment Number)

Your READING ASSIGNMENT for this segment is pages: 116 - 119.

SUPPLEMENTARY NOTES:

In the previous segment, the concept of negative numbers was introduced. It now becomes necessary to set down rules for operating with them.

The definition of the sum of two directed numbers can best be understood by referring to the NUMBER LINE. (See figure below.)



Let us imagine ourselves standing at the origin, facing in the positive direction, and let us interpret each signed number as a marching order. The directed number $^{+}3$ shall mean "march forward 3 units", while the directed number $^{-}3$ shall mean "march backward 3 units." Zero shall mean "do not march."

The plus sign for addition shall mean "follow one order with the next order." The number attached to the point at which we arrive after carrying out all the marching orders is called the sum of the given directed numbers.

You will now be asked a series of questions to emphasize the more important points.

Please go on to page 135
1 .

Question 1

Apply your knowledge of addition on the NUMBER LINE and select the letter which correctly answers this question:

What kind of a number is the sum of two positive numbers?

(A) A negative number.

(B) Sometimes a negative and sometimes a positive number.

(C) Always a positive number.

(D) Sometimes a negative number and sometimes zero.

Did you start by drawing a NUMBER LINE? You should have. Note that in this question, you are not starting at the point 0, but at $^{+}11$. That is, you are eleven units to the right of 0 and you are following commands from this starting position.

Please return to page 146/2 and try this question again.

136
1

We don't agree. There are many cases where it's true. Let us give you a hint. What could you add to $^{-}7$ to get $^{-}10$? Note that $^{-}10$ is less than $^{-}7$.

Can you generalize this situation? As you go further to the left of any point on the NUMBER LINE, the numbers get smaller.

Please return to page 143
2 and try this question again.

136
2

Don't let the letter A confuse you. Draw a NUMBER LINE and designate the point ^{-}A somewhere to the left of the point labeled 0 . Now, consider the other two trips. How far east of ^{-}A will you be after taking a 15 mile trip east and 6 mile trip west?

Please return to page 152
2 and try this question again.

The graph states that all numbers from $^{+}4$ down are in the solution set of the open sentence.

The graph shows this by using the solid circle at $^{+}4$.

If we substitute $^{+}4$ for t in the open sentence, we get

$$2 ({}^{+}4) + 3$$

which has the same value as

$$2 (4) + 3$$

This equals $8 + 3 = 11$

so that this value checks. Now, if t has a smaller value; two times t will have a smaller value than 8 and, therefore, the total when you add 3 will be less than 11 .

Therefore, this graph indicates the solution set correctly.

Please proceed to question 19 below.

Question 19

The temperature at a certain town is never as high as 110° above zero, and its lowest value is 12° below zero. Apply your knowledge to find the choice which describes the domain of the temperature, t .

(A) $^{-}12 < t < {}^{+}110$

(B) $^{-}12 \leq t \leq {}^{+}110$

(C) $^{-}12 < t \leq {}^{+}110$

(D) $^{-}12 \leq t < {}^{+}110$

138
1

One exception is enough to disprove a statement in mathematics.

Consider the sum of

$$({}^{+}8) + ({}^{-}7)$$

The sum of a positive and negative number is not always zero.

When is such a sum equal to zero?

It occurs when the numbers are of the same magnitude; as for example,

$$({}^{+}12) + ({}^{-}12)$$

Please return to page 126/2 and try this question again.

138
2

We don't agree. The letter " x " may have caused you a little difficulty. It usually is helpful to give yourself a similar example with numbers. Suppose that you are at the point $^{+}8$ on the NUMBER LINE, and you took a trip of two miles west; at what point would you arrive?

Yes $({}^{+}8) + ({}^{-}2)$

Now if you analyze what you did with the number 2 , you can do the same with the " x " .

Please return to page 131/2 and try this question again.

The open sentence in this choice states that Harry's marks were between 80 and 100 , with both end values included.

Since the problem specified that he did receive a mark of 80 , and since the highest mark he could have received (on a perfect paper) is 100 , this choice is correct.

Note that we cannot be sure that he did receive 100 , but he might have . Then, in the absence of any other information, we must include the 100 in the domain.

You have now finished Segment 3 . Hand in the PUNCH CARD. You should have entered in your NOTEBOOK the following:

(1) Every number is a directed number and has a position on the NUMBER LINE. If there is no sign written in front, it is taken as plus.

(2) Every two numbers are related in that either
   (a) they are equal
   OR
   (b) one is smaller than the other.

(3) A point on the NUMBER LINE has both a distance from zero and a direction assigned to it.

(4) The scale on the NUMBER LINE must be indicated and the units must be evenly spaced.

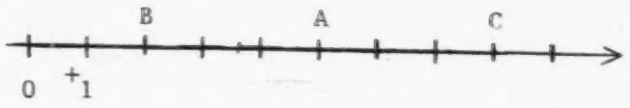
(5) The directed value of the numbers increases as the points move from left to right along the LINE.

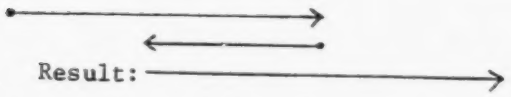
(6) The graph line on the NUMBER LINE includes all of the numbers covered by that line. If the end is a closed circle, then it does include that number.

You should be able to complete the following problems from your HOMEWORK ASSIGNMENT:
Number 4 - Problems 9 - 12.

$\frac{140}{1}$

Very good. You made the correct choice. We draw a NUMBER LINE and follow the commands.





5 units forward brings us to point A , 3 units backward brings us to point B , and 6 units forward brings us to point C . Hence, $^{+}8$ is the correct answer.

Note: It is only necessary to indicate the value of one unit on the scale. Here it equals 1 . Of course, all the marks must be evenly spaced.

Please go on to question 5 below.

$\frac{140}{2}$

Question 5

Apply your knowledge and select the letter which designates the point on the NUMBER LINE that you will arrive at after following these directions: You are at zero, facing right; march backward 7 units then march forward 4 units, and then march backward 9 units.

(A) $^{-}11$

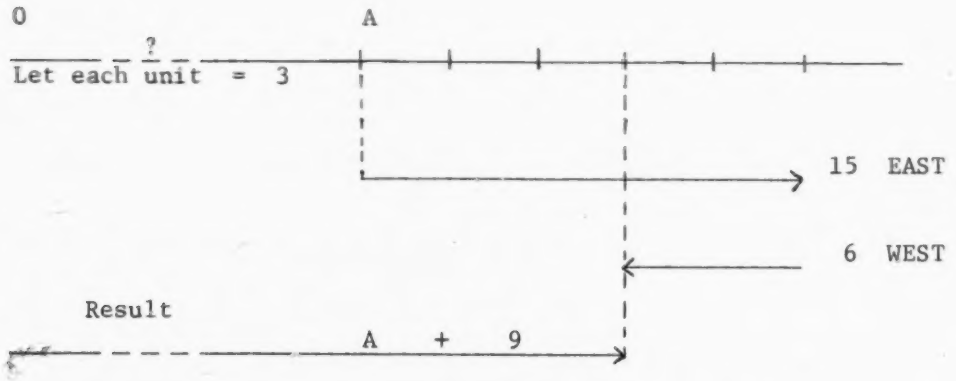
(B) $^{-}12$

(C) $^{-}9$

(D) $^{+}3$

Very good. You made the correct choice. We start at some point ^{-}A on the NUMBER LINE. After making a trip of 15 miles east and 6 miles west, we have actually gone 9 miles east. Hence, we have arrived at the point

$$({}^{-}A) + ({}^{+}9)$$



Please go on to question 9 below.

Question 9

Apply the principles of graphing and find the following sum by using the NUMBER LINE:

$$({}^{+}4) + ({}^{-}9) + ({}^{-}11)$$

Select the letter next to the correct answer.

(A) $^{+}16$ (C) $^{-}2$

(B) $^{-}16$ (D) None of these.

142
1

Surely, you do not mean $3x$. The expression $3x$ means 3 times x . Note that we are adding on the NUMBER LINE and not multiplying.

Please return to page 131
2 and try this question again.

142
2

Although it is usual to have the unit division equal 1 , when the numbers are large either of two situations will develop.

(1) The line will be much too long to manage if the units are distinguishable; or,

(2) if the line is a convenient length, say 3 inches or so, the units would be too tiny to be usable.

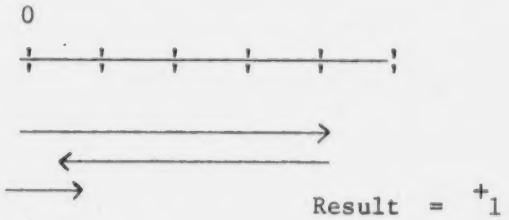
In this problem, the domain runs from $^{-}40$ to $^{+}30$. A total distance of 70 units or about 23 units per inch. This would be much too compact.

Please return to page 163
2 and reconsider the problem.

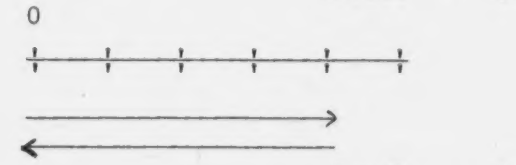
Very good. You made the correct choice. The sum of a positive and negative number can be positive, negative or zero.

The following examples illustrate the various possibilities:

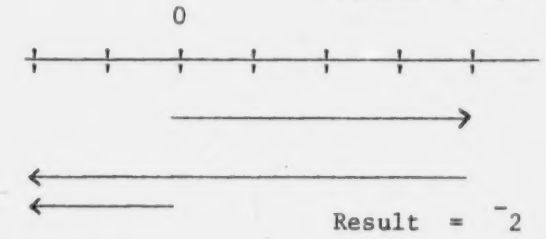
$({}^{+}4) + ({}^{-}3) = ({}^{+}1)$



$({}^{+}4) + (-4) = 0$



$({}^{+}4) + ({}^{-}6) = ({}^{-}2)$



Please go on to question 3 below.

Question 3

Apply your knowledge of addition on the NUMBER LINE and select the letter which correctly answers this question:

When can the sum of two directed numbers be less than both of them?

(A) Never

(B) When one of the numbers is zero.

(C) When both numbers are negative.

(D) When one number is positive and one is negative.

$\frac{144}{1}$

Please note that the sum of

$$\frac{-7}{2} \quad \text{and} \quad -3$$

is greater in magnitude than 6 . Hence, you should end up to the left of the origin, 0 .

Return to page $\frac{154}{2}$ and try this question again.

$\frac{144}{2}$

Normally it is a good practice to let each unit equal 1 , for in most of the problems of this sort you are given small numbers which are whole numbers. However, in some problems, to acquaint you with other types of numbers and the problems associated with them, you are given fractions or decimals. Now when this happens you should adjust your scale to allow those numbers to be graphed easily without any " estimation " of their position on the line.

Return to page $\frac{169}{2}$ and reconsider this question.

There is something to say in having the NUMBER LINE symmetric, but it is not necessary, especially when the positive portion of the line is not used at all. A more serious criticism can be made of the limit on the negative side. A glance at the problem shows that the result will be at the point $^{-}20$. That point certainly should be on the line and labeled.

Return to page 158/2 and reconsider this question.

By this time, you should realize that although any unit will be usable some are better than others.

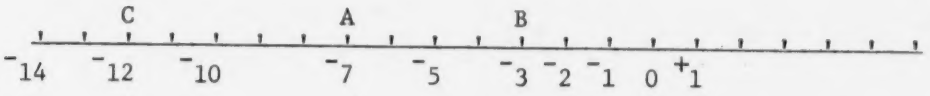
In general, we try to avoid having to estimate the position of any point. Therefore, we choose the smallest unit necessary to achieve this.

Return to page 160/2 and reconsider this question.

146
1

Very good. You made the correct choice. We begin by drawing a NUMBER LINE and mark off units to the right and to the left of the point labeled 0 .

Thus,



Now, 7 units backward brings us to point A . Four units forward brings us to point B , and 9 units backward brings us to point C . Hence, $^{-}12$ is the correct answer.

Please go on to question 6 below.

146
2

Question 6

Apply your knowledge and select the letter which designates the point on the NUMBER LINE that you will arrive at after following these directions:

You are at $^{+}11$ facing right; march backward 7 units, and then march backward 9 units, and then march forward 18 units.

(A) 0

(B) $^{+}11$

(C) $^{+}17$

(D) $^{+}13$

Many mistakes are made in Algebra because of a careless reading of the problem.

Note that we are not starting at the point 0 , but at the point $^{+}8$. Check this carefully.

Return to page $\frac{131}{2}$ and try this question again.

You probably noticed that two of the numbers given are multiples of 15 . However, the 40 is not a multiple of 15 . If you used a unit of 15 , the line would be only 2 units in the positive direction; and 2 and $\frac{2}{3}$ units in the other direction.

It is wise to avoid having to divide units into anything but halves. Any other fraction is quite difficult to handle. Try a different scale.

Return to page $\frac{163}{2}$ and try this question again.

148/1

Did you draw a NUMBER LINE and follow the instructions of this question?

This problem gives you the following marching orders:

Start at $^{+}4$, march 9 units backward, and then, march 11 units backward.

Please return to page 141/2 and try this question again.

148/2

You are disregarding direction. Note that you are starting at the point 6 units to the right of 0 , and then marching backward.

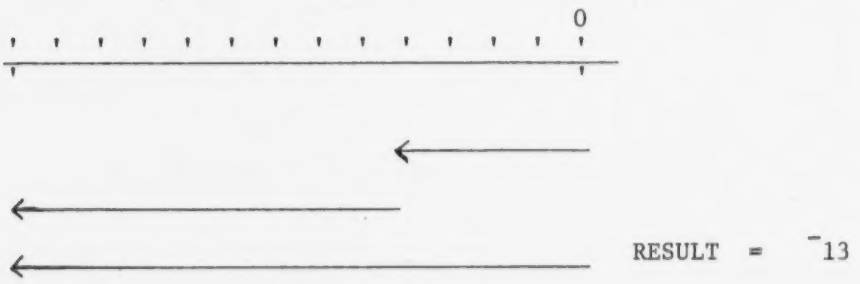
Please return to page 154/2 and try this question again.

Very good. You made the correct choice.

Since we start with a negative number and then move to the left on the NUMBER LINE, when we add a negative the result will still be further left than either one of the two numbers used.

For example, the sum

$$(\ ^{-}4\ ) + (\ ^{-}9\ ) = \ ^{-}13$$



The number $^{-}13$ is smaller than either $^{-}4$ or $^{-}9$.

As we go further to the left of any point on the NUMBER LINE, the numbers get smaller.

Please go on to question 4 below.

Question 4

Apply your knowledge and select the letter which designates the point on the NUMBER LINE that you will arrive at after these directions:

You are at zero facing right towards the positive numbers: march 5 units forward, and then march 3 units back, and then march 6 units forward.

(A) $^{+}11$

(B) $^{+}8$

(C) $^{+}4$

(D) $^{-}4$

150
1

Good, you realized that since the numbers you had to graph were decimals (tenths, actually) you should have a basic unit in tenths.

But why did you choose .2 ? You see, when you have to graph numbers like $^{+}0.8$ or $^{-}1.2$, a unit of .2 will work fine; but won't you have to estimate the actual position for $^{+}1.5$ and $^{-}2.3$?

Now it's true that sometimes you can't avoid having to estimate position, but here with such small numbers you can afford to have a very fine scale.

Please return to page 169/2 and try this problem again.

150
2

The simplest way to analyze a problem of this type is to construct a NUMBER LINE. Determine an appropriate scale and an appropriate domain. Then the "direction" of the motions involved must be established. If one motion is "against" another, then one will be designated as a positive direction and the other as a negative direction.

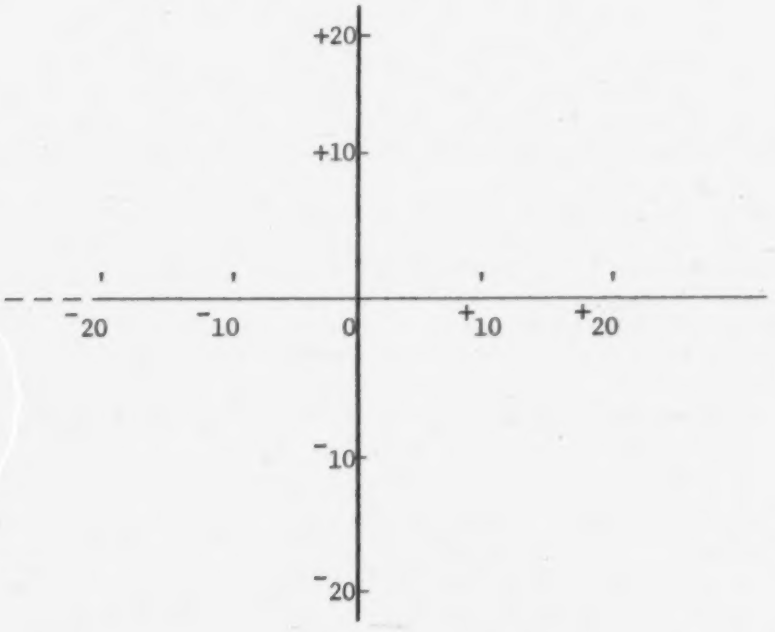
Please return to page 156/2 and try this question again.

It is not wrong to have the NUMBER LINE symmetric, but it does crowd the scale if half of the line is unused, since it does take up space.

Furthermore, it is not wise to end the line at the very point where the problem ends. It should extend a little beyond to indicate that the line continues indefinitely.

Please return to page 158/2 and try this question again.

We do not agree. Although the NUMBER LINE is usually drawn horizontally, and a thermometer tube is vertical, we still can think of a thermometer as a NUMBER LINE. The units above zero are positive, and the units below zero are negative.



Please return to page 167/2 and try this question again.

152
1

Very good. You made the correct choice. We begin at point $^{+}8$ and take a trip of x miles west. We arrive at the point

$$({}^{+}8) + ({}^{-}x)$$

Then we take a trip of 5 miles east and arrive at

$$({}^{+}8) + ({}^{-}x) + ({}^{+}5)$$

We end up at the point

$$ {}^{+}13 + ({}^{-}x)$$

It should be noted that the order of the trip is not important, since the addition of signed numbers is commutative; that is, we can reorder

$$({}^{+}8) + ({}^{-}x) + ({}^{+}5)$$

to

$$({}^{+}8 + {}^{+}5) + ({}^{-}x)$$

which becomes

$$({}^{+}13) + ({}^{-}x)$$

Please go on to question 8 below.

152
2

Question 8

Apply your knowledge and select the letter which designates the point on the NUMBER LINE that you will arrive at after following these directions:

You are at a point ^{-}A facing east. Take a trip of 15 miles and then a trip of 6 miles west.

(A) $({}^{-}A) + ({}^{+}9)$

(B) $ {}^{+}A + ({}^{-}9)$

(C) $ {}^{+}21 + ({}^{-}A)$

(D) $ {}^{+}A + ({}^{-}21)$

You know, theoretically, you are right. Any scale can be used to represent numbers on the NUMBER LINE. However, we must be practical. To take an extreme case, would you let each unit equal 100 in this problem? Can you imagine all the trouble you'd have trying to decide just where to put $\bar{}15$?

And on the other extreme, would you pick 0.1 as a unit length in this problem? Why, for $\bar{}40$ you would need 400 units.

So as you can see, it does make a difference.
The choice depends on the numbers involved in the problem.

Please return to page 163/2 and reconsider this problem.

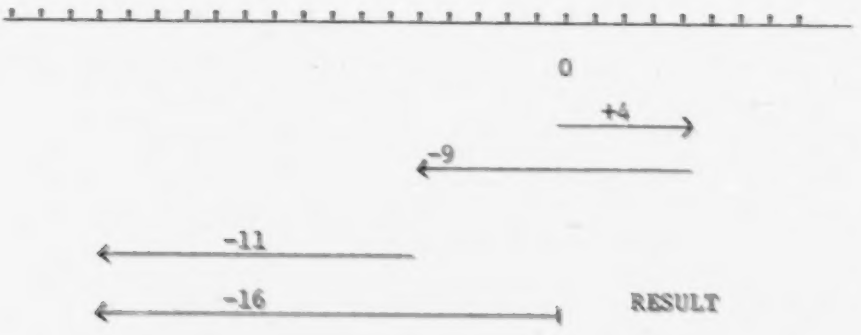
We don't agree.

The purchase price of a share of stock is \$15 .
A stock going up a certain amount is equivalent to a move to the right on the NUMBER LINE by that amount. Similarly, a stock going down a certain amount is equivalent to a move to the left. Furthermore, be more careful in adding all the amounts of this question. Choose a proper unit for your scale. You note that the problem deals in fractions of dollars, and the answer is in dollars and cents.

Please return to page 161/2 and try this question again.

154
1

Very good. You made the correct choice. Let's review the procedure. We start by drawing a NUMBER LINE. The first number, $^{+}4$ tells us that we are 4 units to the right of 0 . Then we are ordered to march 9 units backward, and again, 11 units backward. We arrive at the point $^{-}16$ on the NUMBER LINE.



Please go on to question 10 below.

154
2

Question 10

Apply the concept of addition on the NUMBER LINE and find the following sum:

$$({}^{+}6) + ({}^{-}\frac{7}{2}) + ({}^{-}3)$$

Select the letter next to the correct answer.

(A) $^{-}\frac{1}{2}$

(B) $^{+}\frac{1}{2}$

(C) $^{+}3\frac{1}{2}$

(D) $^{-}3\frac{1}{2}$

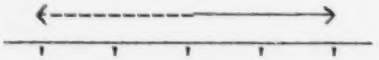
Yes, any unit is better than none, but the important point is to have a workable unit that will allow the most precision in the graphing of the numbers, and, you will agree, some units are better than others for certain problems.

Consider the numbers involved and then choose an appropriate unit scale.

Please return to page 169/2 and reconsider the problem.

Think of the point labeled 0 on the NUMBER LINE as a starting point. Any point on the NUMBER LINE on one side of 0 is matched by a point on the opposite side having the same number of units from 0 . The matching number is called the opposite of the original number.

e. g.



-2 is the opposite of 2

2 is the opposite of -2

or in general,

-N is the opposite of N

and conversely,

N is the opposite of -N

Please return to page 171/2 and try this question again.

156
1

Good. Since the purpose of the scale on the NUMBER LINE is to illustrate accurately the position of each number involved in the problem, it is considered good practice to choose the smallest scale necessary to allow this. Since the smallest unit in this group of fractions involves "eighths" we choose the basic unit to be

$$\frac{1}{8}$$

Please proceed to question 15 below.

156
2

Question 15

Apply the NUMBER LINE graph to analyze this problem and select the letter next to the correct answer.

A motorboat which has a top speed of 12 miles per hour is cruising against a current whose speed is 3 miles per hour; how fast is the boat actually moving?

(A) 8 miles per hour

(B) 15 miles per hour

(C) 9 miles per hour

(D) 4 miles per hour

157/1

Construct a NUMBER LINE and use 1 pound as the basic unit. Think of the weight of the man as $^{+}180$. Losing weight can be considered as moving to the left on the NUMBER LINE. Gaining weight can be considered as moving to the right on the NUMBER LINE.

Please return to page 162/2 and try this question again.

157/2

We agree, but the statement "very cold" is not an accurate description. In mathematics, a numerical answer is usually preferred. Furthermore, a temperature of $^{-}30°$ may be considered very cold in your area, but an Eskimo living near the Arctic region may not consider $^{-}30°$ to be very cold at all.

Please return to page 167/2 and try this question again.

Considering the numbers that are given in the problem, your choice of 5 per unit seems to be the best. In the first place, all of the numbers are multiples of 5 and, therefore, you will not run into fractions in graphing the points. Secondly, the largest unit in one direction would require only 8 units, and the largest in the other direction needs only 6 units. Therefore, the total length of 14 units is not too unwieldy or too small. That was a good choice.

Please proceed to question 12 which follows.

Question 12

Apply the principles of NUMBER LINE construction and determine the domain of the portion of the line that you would draw and label to handle the graphing of the following problem:

$$(\ ^{-}10\ ) + (\ ^{-}5\ ) + (\ ^{-}5\ )$$

(A) $^{-}10$ to $^{+}10$

(B) $^{-}20$ to $^{+}20$

(C) $^{-}25$ to 0

(D) $^{-}50$ to $^{+}50$

Could this be a case of careless reading?

In this question you are not starting at the point 0 ; you are starting at the point +6 which is 6 units to the right of point 0 .

Please return to page 154/2 and try this question again.

The "additive inverse" and the "opposite" of a directed number are two expressions meaning the same thing. Thus, the additive inverse of

N is -N

or as a second example, the additive inverse of

(a + b) is - (a + b)

Please return to page 177/2 and try this question again.

160
1

Good. It looks like you understand the problem of choosing a proper scale in constructing a NUMBER LINE.

Since all of the numbers are in decimal form, you are right in choosing a decimal value for your unit. Secondly, since all of the decimals are "tenths" (with no common divisor) it is advisable to choose 0.1 as the unit, for then each of the numbers can be graphed precisely without any estimation of the position on the line.

Please proceed to question 14 below.

160
2

Question 14

If you were directed to show the following addition on the NUMBER LINE, apply the principles of NUMBER LINE construction and show which unit you would choose for the scale.

$$(^{+}2\tfrac{1}{2}) + (^{-}\tfrac{1}{4}) + (^{+}\tfrac{13}{8}) + (^{-}\tfrac{3}{4})$$

(A) Each unit would equal 1

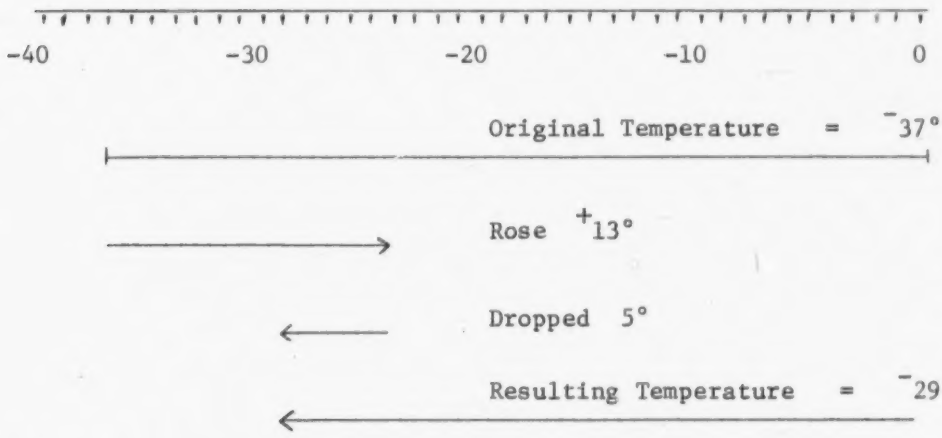
(B) Each unit would equal $\frac{1}{2}$

(C) Each unit would equal $\frac{1}{4}$

(D) Each unit would equal $\frac{1}{8}$

Very good. You made the correct choice. We can think of the thermometer as a NUMBER LINE with zero degrees at the origin, above zero the positive side, and below zero the negative side. Thus, the question reduces to finding the sum of the signed numbers,

$$(-37) + (+13) + (-5) \quad \text{This is equal to } -29 .$$



Please go on to question 18 below.

Question 18

Apply the techniques of NUMBER LINE CONSTRUCTION and select the letter next to the correct answer to the following question: Mr. Y bought a certain stock at \$15 a share. During the week the stock changed as follows: Monday, down $\$2\frac{1}{2}$; Tuesday and Wednesday, up $\$\frac{3}{4}$; Thursday, unchanged; and Friday, up $\$1\frac{1}{2}$. What was the final price of the stock?

(A) \$14.25

(B) \$15.25

(C) \$15.50

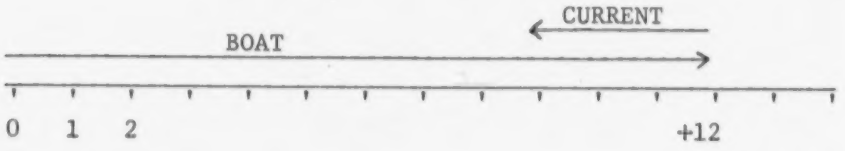
(D) \$14.75

162
1

Very good. You made the correct choice. The motorboat has a top speed of 12 miles per hour. In other words, the boat would cruise at this speed if there was no current. However, the boat is moving <u>against</u> the current, whose speed is 3 miles per hour. Hence, the effective speed of the boat is

+12 + (-3) and this is equal to +9

This can be easily shown as a NUMBER LINE DIAGRAM



Please go on to the next question, below.

162
2

<u>Question 16</u>

Apply the principles of the NUMBER LINE GRAPH to analyze this problem.

Select the letter next to the correct answer to this question:

A man weighing 180 pounds went on a diet and lost 18 pounds.

He then went off the diet and gained 7 pounds.

How much did he weigh at the end?

(A) 177 pounds

(B) 181 pounds

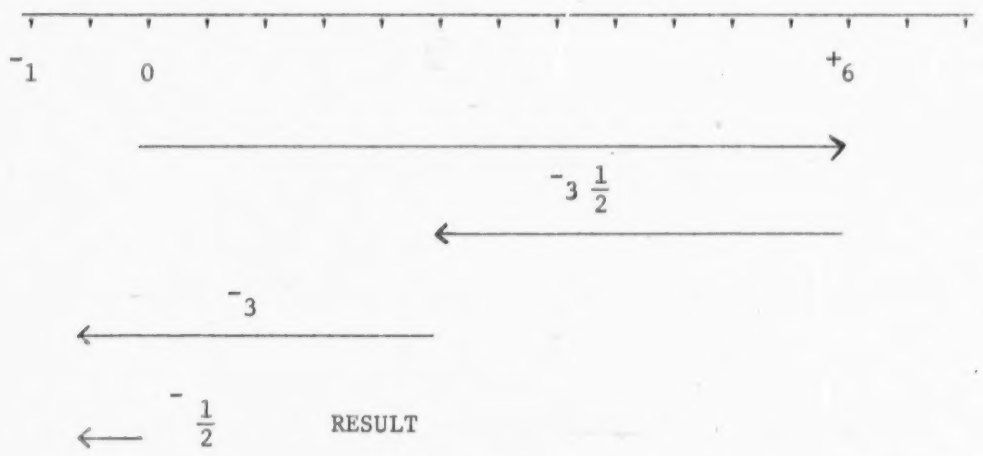
(C) 191 pounds

(D) None of these

Very good. You made the correct choice. We begin at the point 6 units to the right of the point labeled 0 . We march backward

$$\frac{7}{2} = 3\frac{1}{2}$$

units, then march backward 3 more units. We arrive at the point $\frac{1}{2}$ unit to the left of 0 . Hence, $^{-}\frac{1}{2}$ is the correct answer.



Please go on to question 11 below.

Question 11

Apply the principles of NUMBER LINE CONSTRUCTION and determine the scale you would use to be able to represent the following problem on a line of about three inches in length.

$$(\ ^{-}15\ ) + (\ ^{+}30\ ) + (\ ^{-}40\ )$$

(A) Let each unit equal 1

(B) Let each unit equal 15

(C) Let each unit equal 5

(D) It makes no difference, any unit is satisfactory.

164
1

Well, this choice could certainly handle this problem, but weren't you a bit generous in setting the limits so far apart?

The segment of the NUMBER LINE that you chose is actually a hundred units long. The scale marks would have to be very close together; so much so that it would make the scale difficult to read.

Secondly, isn't it wasteful to have the entire positive portion of the line there, but unused?

It is best to adjust the scale to fit each problem.

Please return to page 158/2 and try this problem again.

164
2

You have the right idea. You did choose the opposite of each number given, and you did present them as a sum; but the sum was only an indicated sum. You were asked to choose a number; that is, a single number which is equivalent to the opposite of the single number sum of the original problem.

Please return to page 171/2 and try this question again.

You made the correct choice.

The "additive inverse" of a number is the same as the "opposite" of the number.

There are two methods of finding the additive inverse of a sum:

(1) Find the sum first and then take the opposite:

e.g. (+7 + 19) = (+26) The opposite is -26

(2) Find the opposite of each and then find the sum:

e.g. The opposites of (+7 + 19) are (-7) + (-19) and the sum is -26 .

Please go on to question 3 below.

Question 3

Apply the principle of additive inverses and find the following sum,

$$-(-9) + (-3) + 8$$

Select the letter next to the correct answer.

(A) +14

(B) - 4

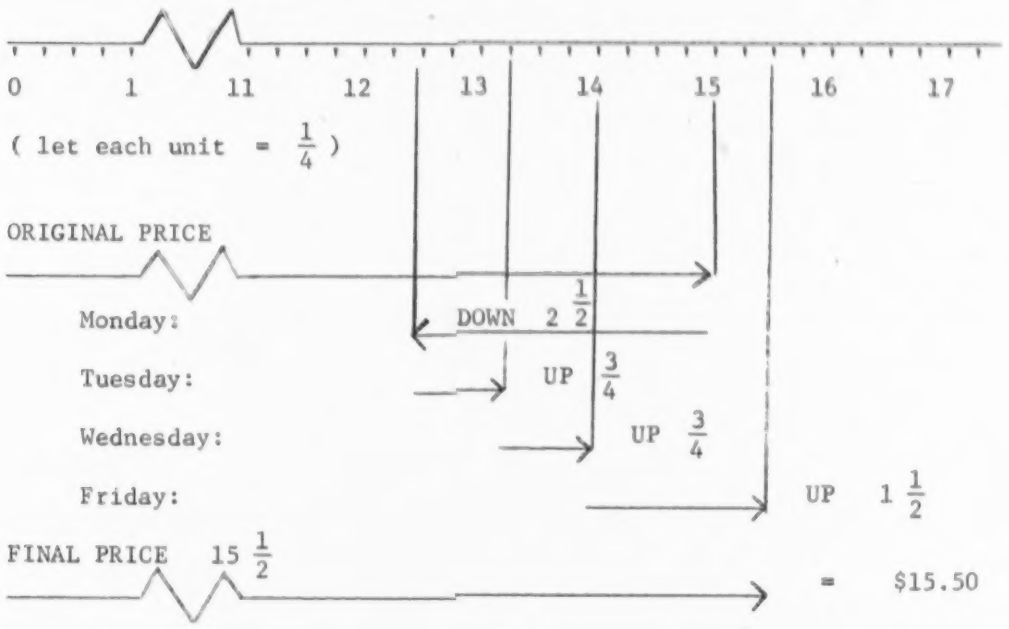
(C) -20

(D) None of these.

Very good. You made the correct choice.

Our first task is to determine the domain of the segment of the NUMBER LINE. It centers around the number 15 and the changes range from $\frac{3}{4}$ to $2\frac{1}{2}$. Therefore, it looks like we will have another "broken line" graph. We'll indicate the zero, but we will skip to the area near 15 for the actual depicting of the operations.

Secondly, the smallest unit of change is in "quarters", therefore, that will indicate that $\frac{1}{4}$ should be our basic unit.



The purchase price of the stock was \$15. This amount can be represented on the NUMBER LINE as $^{+}15$. The stock going down $2\frac{1}{2}$ dollars means that $^{-}2\frac{1}{2}$ should be added to \$15. A stock going up $\frac{3}{4}$ means that $^{+}\frac{3}{4}$ should be added.

Thus, the question reduces to finding the sum of

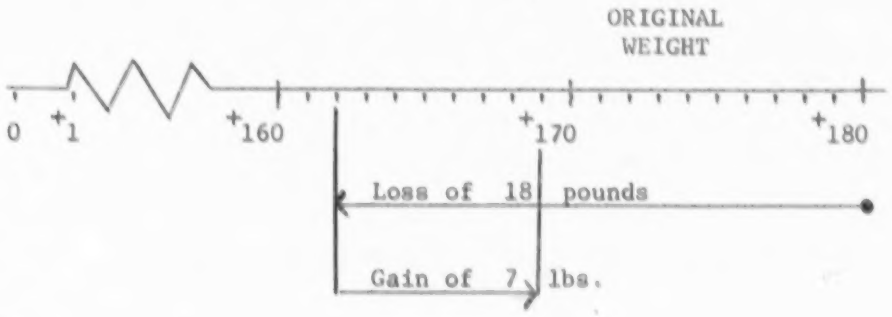
$$^{+}15 + (^{-}2\tfrac{1}{2}) + (^{+}\tfrac{3}{4}) + (^{+}\tfrac{3}{4}) + (^{+}1\tfrac{1}{2})$$

This sum equals $^{+}15.50$.

You have now finished this segment. Hand in your PUNCH CARD.

You should now be able to complete the following problems from your HOMEWORK ASSIGNMENT: Problems 13, 14, 15, and 16.

Yes, "none of these" is the correct answer.
In a problem of this sort, when we resort to the NUMBER LINE to analyze the relationships, we have a problem. On the one hand, we have to use one pound as the unit scale, but although the changes are relatively small, a loss of 18 and a gain of 7 , the original amount is quite large. We, therefore, resort to a device by indicating a "broken" NUMBER LINE as follows:



169 net weight

If a man weighs 180 pounds, we can think of his weight as $^{+}180$ on the NUMBER LINE. A loss of 18 pounds means moving 18 units to the left on the NUMBER LINE. A gain of 7 pounds means moving 7 units to the right on the NUMBER LINE. Thus, we have the sum of

$$^{+}180 + (\,^{-}18\,) + (\,^{+}7\,)$$

This sum equals $^{+}169$.

Please go on to question 17 below.

Question 17

Apply the principles of the NUMBER LINE graph to analyze this problem: During a cold wave, the temperature in the morning was 37° below zero. The temperature rose 13° during the day and then dropped 5° in the evening. What was the temperature in the evening?

(A) $^{-}39°$

(B) $^{-}18°$

(C) Very cold

(D) $^{-}29°$

168/1

We do not agree.

Adding or subtracting 0 does not affect the sum.

Also note that + 0 and - 0 are the same number.

Please return to page 184/2 and try this question again.

168/2

We do not agree.

If you replace the letters by their given numerical values, you get

$$17 + [- (8 + 9)]$$

Remember, when we have parentheses within other symbols of inclusion, we must work from the inside out.

Note that we have to add the opposite of

$$(8 + 9) \text{ to } 17$$

Please return to page 180/2 and try this question again.

Of all the choices this appears to be the best.

Let's examine why we say this. First, the zero point is indicated; on any graph it is essential that this be shown so as to illustrate the true relationship between the size of the quantities involved. Secondly, the $^{-}25$ is beyond the limit of the problem, but not too far removed. This allows for the proper emphasis of the positions of the numbers involved and indicates that they form only a part of the line. Lastly, this particular segment of the NUMBER LINE can accommodate all of the numbers involved in the problem no matter which order is chosen to enter them on the line.

Please proceed to question 13 below.

Question 13

If you were directed to show the following addition on the NUMBER LINE, apply the principles of NUMBER LINE CONSTRUCTION and indicate what unit you would choose for the scale,

$$(\,^{+}15\,) + (\,^{-}2.3\,) + (\,^{-}1.2\,) + (\,^{+}0.8\,)$$

(A) Each unit would equal 0.1

(B) Each unit would equal 1.0

(C) Each unit would equal .2

(D) Any unit will do.

170/1

We do not agree.

Evaluating an expression consisting of letters whose values are given is done by replacing the letters by the given numerical values. After this is done, the indicated operations are performed.

Remember, you need not draw a NUMBER LINE, but mentally refer to it to verify your calculations.

Please return to page 174/2 and try this question again.

170/2

We don't agree. It is considered good form to keep each succeeding substitution in a row directly under the previous line. Then one operation is performed at a time; one to each line.

Did you replace the letters by their given numerical values? you get:

$$a + b - [(-c) + d]$$

$$5 + 3 - [(-7) + 2]$$

Recall that the additive inverse of the sum of two numbers is the sum of their additive inverses.

Please continue.

Please return to page 176/2 and try this question again.

VOLUME 4 SEGMENT 5 begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS 48 and 50 2 0 (Sequence Number)

54 and 56 0 4 (Type of Punch Card)

60 and 62 0 4 (Volume Number)

66 and 68 0 5 (Segment Number)

Your READING ASSIGNMENT for this segment is pages 120 - 123 .

SUPPLEMENTARY NOTES:

The NUMBER LINE was introduced to provide a graphic interpretation of the meaning of positive and negative numbers. It was also used as an aid in the understanding of the operations of addition and subtraction with signed numbers. It was not intended that you always rely on the NUMBER LINE to perform these operations, but that you have it as a concrete example to fall back on to support the quicker abstract methods that we are now going to develop.

Since you have no doubt observed that:
addition of a positive number produces the same result as adding the number; and the addition of a negative number produces the same result as subtracting the number;
we are going to make the following simplifications in notation:

$+ N$ will replace $+ {}^{+}N$

and

$- N$ will replace $+ {}^{-}N$

and $+ N$ and $- N$ will be referred to as "opposites" or "additive inverses" . You will now be asked a a series of questions to draw your attention to the more important points.

Question 1

Recognize the number that is the opposite of the following sum:

$$(+8) + (-38) + (-20)$$

Select the letter next to the correct answer.

(A) -50

(B) +50

(C) -66

(D) (-8) + (+38) + (+20)

172
1

We do not agree.

You must consciously apply the rules.

Please note that the opposite of

$-a$ (that is $-(-a)$ is a)

Thus, for example, $-(-7)$ can be replaced by $+7$

Please return to page 165 (2) and try this question again.

172
2

We do not agree.

It is important that you understand clearly the notion of the absolute value of a number. Read the next sentence carefully. The absolute value of a signed number is the number that remains when the sign is removed. Thus, the absolute value of -9 written

$$|-9| \text{ is } 9$$

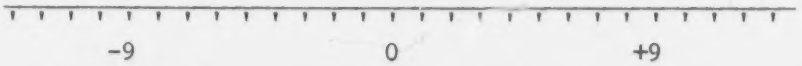
Notice that we removed the minus sign. The absolute value of

$+9$

written $|+9| = 9$

Again we can remove the sign before the number, since $+9$ and 9 stand for the same number.

Referring to the NUMBER LINE the ABSOLUTE VALUE of a "signed number" is its actual distance from the zero point without regard to its direction.



Both numbers have the absolute value " 9 "

Please return to page 182 (2) and try this question again.

Here is a very compact rule:
The opposite of the sum of two numbers is the sum of their opposites.
In proto-type form this would be

The opposite of (a + b)

is -(a + b) which becomes

(-a) + (-b)

Now this basic relationship holds even if one of the terms is itself negative.

Consider the sum:	(5 - 7)	Alternate Method (5 - 7)
The opposite is:	-(5 - 7)	
		the sum is -2
This becomes by the rule:	-(+5) + -(-7)	the opposite is -(-2)
	-5 + 7	which equals +2
The sum is	+ 2	

Use either technique.

Please return to page 179/2 and try the problem again.

We don't agree. What is the absolute value of -12 ?
Remember, the absolute value of a signed number is the number with the sign removed.
Thus, $|-12| = 12$

Do not confuse the rule that

$$-(-a) = a$$

with the expression $-|-a|$ which equals $-a$.

Please return to page 181/2 and try this question again.

IV

174
1

Very good.

You made the correct choice.

Note that +0 and -0 are the same and do not affect the sum.

Thus, we have to find the sum of

-9.2 and -6.4

the answer is -15.6

Please go on to question 6 below.

174
2

Question 6

Apply your knowledge and evaluate the expression

$$x + (-y) + z$$

if $x = 7$

$y = 9$

$z = 11$

Select the letter next to the correct answer.

(A) -13

(B) - 9

(C) + 9

(D) -11

Recall your work with parentheses and brackets. We always simplify by removing the innermost symbols first. Absolute value symbols can also be considered as symbols of inclusion. It is important to note that while parentheses ( ) , brackets [ ] . and braces $\{\ \}$ all have the same purpose of isolating as one group whatever is inside them, the two vertical straight lines $|\ |$ have an additional purpose, that of converting the enclosed signed number to its absolute value.

Now,

$$|8| = 8$$

and

$$|-8| = 8$$

We, therefore, have to evaluate

$$-\ \{8 + 8\}$$

Please continue.

Please return to page 187/2 and try this question again.

Imagine that you are on the NUMBER LINE, 6 units to the left of the point labeled 0 . This corresponds to the term (-6) in the given equation. Ask yourself the question: How many units to the right must I march to reach the point +5 ?

Please return to page 194/2 and try this question again.

176
1

Very good. You made the correct choice. After replacing the letters by their given values, we get

$17 + [-(8 + 9)]$	=	We use the rule:
$17 + [-8 + (-9)]$	=	$-(a + b) = -a + (-b)$
$17 + [-17]$	=	Since:
		$a + (-a) = 0$
		for all a.

this then = 0

Please go on to question 8 below.

176
2

Question 8

Apply your knowledge and evaluate the expression,

$$(a + b) - [(-c) + d]$$

if $a = 5$

$b = 3$

$c = 7$

$d = 2$

Select the letter next to the correct answer.

(A) 13 (C) 17

(B) -3 (D) -1

Very good. You made the correct choice. The opposite of any directed number " a " is the directed number whose sum with " a " equals zero.

That is, a and -a are opposites since

$$(+a) + (-a) = 0$$

Thus, the sum of

$$(+8) + (-38) + (-20) = -50 \text{ and}$$

$$(-50) + (+50) = 0$$

Hence, +50 is the opposite of -50 .

Please go on to question 2 below.

Question 2

Apply your knowledge and find the additive inverse of

$$(+7 + 19)$$

Select the letter next to the correct answer.

(A) +26

(B) -12

(C) -26

(D) +12

178
1

Your choice is incorrect. One of the letters does have the correct answer next to it. Stay awhile; let us do a similar question together. Evaluate the expression,

$$|-15| + |-5| - |18|$$

Now keep in mind that the absolute value of a signed number is the number that remains when the sign is removed. Thus,

$$|-15| = 15$$
$$|-5| = 5$$
$$|+18| = 18$$

We now have,

$$15 + 5 - 18 = 2$$

Please return to page 182/2 and try this question again.

178
2

We don't agree. An easy way to discover that your choice is incorrect is to replace the variable by the number that you chose, and see if you get a true statement.

Make believe that you are at +4 on the NUMBER LINE. What marching order must you execute in order to arrive at -16 on the NUMBER LINE?

Please return to page 188/2 and try this question again.

Very good. You made the correct choice.

In Algebra, each expression is systematically replaced by an equivalent expression which is constructed through the application of accepted definitions or rules.

Expression:	Authority:
$-(-9) + (-3) + 8$	$[\ -(-a) = a$
$9 + (-3) + 8$	[COMMUTATIVE LAW
$(9 + 8) + (-3)$	$[\ +(-a) = -a$
$17 \quad -3$	
14	

Please go on to question 4 below.

Question 4

Apply your knowledge and find the following sum,

$$-(4 - 21)$$

Select the letter next to the correct answer.

(A) -25

(B) +25

(C) -17

(D) +17

180
1

Very good. You made the correct choice. To evaluate an expression with letters, we first replace the letters by their given numerical values. Thus, we get,

$$7 + (-9) + 11$$

This sum is equal to

$$+9$$

Please go on to the next question.

180
2

Question 7

Apply your knowledge and evaluate the expression

$$p + [- (q + r)]$$

if $p = 17$

$q = 8$

$r = 9$

Select the letter next to the correct answer.

(A) 34

(B) 0

(C) -17

(D) +17

Very good. You made the correct choice.

First we note that

$$|8| = 8$$

and $$|-8| = 8$$

since the absolute value of a positive number is the number itself, and the absolute value of a negative number is the opposite of the same number.

That is, $|+a| = a$

$|-a| = a$ (the opposite of $-a$)

NOTE: the simpler way of handling such numbers is to merely consider the number with the sign ignored or dropped. Then both

$|+a|$ and $|-a| = a$

We, therefore, have

$$- (8 + 8)$$

Using the rule that

$$- (a + b) = - a + (- b)$$

we get

$$- (8 + 8) = - 8 + (- 8)$$

$$= -16$$

Please go on to question 11 below.

Question 11

Apply your knowledge and evaluate the expression,

$$|16| - |-12| + |2 - 8|$$

Select the letter next to the correct answer.

(A) 34 (C) -2

(B) 10 (D) 22

182
1

Very good. You made the correct choice.

We first replace the letters by their given values.

Let $a = 5$

$b = 3$

$c = 7$

$d = 2$

$a + b - [(-c) + d]$

We substitute $5 + 3 - [(-7) + 2]$

the given values $8 - [-5]$ Note: $-[-x] = +x$

$8 + 5$

hence 13

Please go on to question 9 below.

182
2

Question 9

Apply your knowledge and evaluate the expression,

$$|5| + |13| - |-7|$$

Select the letter next to the correct answer.

(A) 25

(B) 11

(C) 9

(D) None of these.

Did you guess?

Let us make believe that your choice, -1 , is correct.

Replacing the letter x by -1 we get

$$-1 + (-6) = +5$$

Clearly, the above is not a true statement.

You must check your choice before you say that you have the correct answer.

Please return to page 194/2 and try this question again.

Suppose that we replace x by some number, say 3 .

Then,

$$|x| = |3|$$

$$|x| = 3$$

Now, let us replace

x by -3

we get

$$|x| = |-3|$$

$$|x| = 3$$

Thus, the absolute value of x remains unchanged if we replace x by its opposite.

Does this help you to answer this question.

Please return to page 195/2 and try this question again.

184
1

Very good. You made the correct choice.

The rule for finding the opposite or the additive inverse of a sum is to take the sum of their opposites.

Thus, the opposite of +4 is -4 , and the opposite of -21 is +21 .

The sum of -4 and + 21 is +17.

Please go on to question 5 below.

184
2

Question 5

Apply your knowledge and find the following sum:

$$-9.2 + (-0) + (-6.4)$$

Select the letter next to the correct answer.

(A) -15.6

(B) - 6.4

(C) - 2.6

(D) -15.2

One way to answer this question is to determine what trip you must take on the number line to arrive at -16 ; that is, 16 units to the left of the point 0 , if you start 4 units to the right of 0 . Yes, you must make a trip of 20 units to the left of 0 . Hence, the answer is -20.

Since this is a member of the set you chose, you are correct.

Please go on to question 14 below.

Question 14

Apply your knowledge and find a positive or negative number to replace the variable which will make the following open sentence a true statement:

$$-4 + g = -16$$

Select the letter next to the set that contains the correct answer.

(A) $\{0 , 4 , 8 , 12\}$

(B) $\{-4 , -8 , -12 , -16\}$

(C) $\{14 , 16 , 18 , 20\}$

(D) $\{-14 , -16 , -18 , -20\}$

186
1

The absolute value of a number is always a positive number. Thus,

$$|y| > 0$$

and $$|4| = 4$$

Do you now see why your choice is incorrect?

Please return to page 198/2 and try this question again.

186
2

One of the proper methods for solving such an equation is to start by removing the absolute value symbols.

Thus,

$$|b| - 18 = |-20|$$

is equivalent to

$$b - 18 = +20$$

Please continue by using the equation property that applies.

Please return to page 196/2 and try this question again.

Very good. You made the correct choice.

The absolute value of a positive number is the number itself; the absolute value of a megative number is the opposite of the number, which is again positive.

Therefore, we proceed in this fashion:

						Since:		
Given:	$\|5\|$	+	$\|13\|$	-	$\|-7\|$	$\|5\|$	=	5
therefore,	5	+	13	-	7	$\|13\|$	=	13
			18	-	7	$\|-7\|$	=	7
					11			

Please go on to question 10 below.

Question 10

Apply your knowledge and evaluate the expression

$$-\left\{|8| + |-8|\right\}$$

Select the letter next to the correct answer.

(A) -16

(B) 0

(C) 16

(D) -0

188
1

Very good. You made the correct choice.

You probably asked yourself the following question:

What number must be added to (-6) in order to obtain +5 ?

Or, you could have reasoned thus:

I owe $6 ; how much more must I receive to pay off the $6 and still have $5 left?

The answer is $11 .

Please go on to question 1 below.

188
2

Question 1

Apply your knowledge and find a positive or negative number to replace the variable which will make the following open sentence a true statement?

$$4 + f = -16$$

Select the letter next to the set that contains the correct answer.

(A) $\{0 , 4 , 8 , 12\}$

(B) $\{-4 , -8 , -12 , -16\}$

(C) $\{14 , 16 , 18 , 20\}$

(D) $\{-14 , -16 , -18 , -20\}$

The expression

$$|2 - 8|$$

does not equal

$$-6$$

it is equal to

$$|-6|$$

Remember

$$|2 - 8|$$

is not the same as

$$(2 - 8)$$

Please reconsider.

Please return to page 181/2 and try this question again.

Yes, but is 5 the only member of the solution set? Recall the fact that

$$|a| = a$$

if $a > 0$

and $|a| = -a$

if $a < 0$

Or in other words,

$$|7| = 7$$

$$|-7| = 7$$

Please return to page 195/2 and try this question again.

190
1

We don't agree. It helps to refer to the NUMBER LINE.

The number -4 is the point on the NUMBER LINE 4 units to the left of the point labeled 0 .

You must now add some number, denoted by the letter g .
After adding this number, you should be 16 units to the left of 0 .
What is the number?

Please return to page 185/2 and try this question again.

190
2

We don't agree. Suppose for a moment that your choice is correct. Then we can check your answer by substituting it in the original equation:

$$b - |18| = |-20|$$

$$-2 - |18| \quad \text{should equal} \quad |-20|$$

$$-2 - 18 = 20$$

It looks like you forgot that

$$|-a| = a$$

Please return to page 196/2 and try this question again.

Too bad! It looks like you analysed the problem correctly. You realized that no number has an absolute value that can be added to the absolute value of 4 to get 3 .

But, does 0 mean "no number" or does it rather mean the number zero?

Now you know

$$0 + 4 \neq 3$$

Perhaps you'd like to answer the problem again.

Please return to page 198/2 and reconsider question 16 .

One procedure is to pick a value for x from the domain, substitute, and then evaluate each by removing the absolute value signs.

When these are related to the NUMBER LINE, you will have your answer.

Please return to page 193/2 and attack the problem.

192/1

Do you know what we mean when we say that a set is closed under a specific operation?

Consider, for example, the meaning of the CLOSURE PROPERTY OF ADDITION. For every number a and every number b in a given set, (a + b) is also a member of this set.

Closure under the operation of absolute value means:

For every element in the set, the absolute value of that element is also in the set.

Examine each of the given sets, element by element, and see if each element of a set also has its absolute value as an element of the set.

Please return to page 197/2 and try this question again.

192/2

Suppose we replace the expression

$$x - 1$$

by a single letter, say y . The given equation becomes

$$|y| = 3$$

What values may the letter y have in order for its absolute value to be equal to 3 ?

Then let those values equal $x - 1$

and solve for x

Don't forget to check each member of the set.

Please return to page 199/2 and try this question again.

Very good. You made the correct choice.

We first rewrite the given equation with the absolute value symbols removed.

We get

$$b - |18| = |-20| \text{ since } |a| = a$$

$$b \quad 18 = 20 \text{ and } |-a| = a$$

Using the ADDITION PROPERTY OF EQUALITY, we obtain the equivalent equation,

$$b - 18 + 18 = 20 + 18$$

and $$b = 38$$

Please go on to the next question.

Question 18

Use the definition of absolute value as applied to the NUMBER LINE and arrange the following expressions in order from the smallest to the largest. Choose the letter next to the proper sequence where x is a member of the set of positive integers.

$$P = |-x| \quad Q = |-2x| \quad R = -|x| \quad S = -2|x|$$

(A) S Q R P

(B) S R P Q

(C) R Q S P

(D) P R Q S

194
1

Very good. You made the correct choice.

We have to evaluate the expression

$$|16| - |-12| + |2 - 8|$$

Now $|16| = 16$

$|-12| = 12$

and 1 $|2 - 8| = |-6|$

$= 6$

Thus, we have to find the value of

$$16 - 12 + 6$$

and this is equal to 10

Please go on to question 12 below.

194
2

Question 12

Apply your knowledge and find a positive or negative number to replace the variable which will make the following open sentence a true statement:

$$x + (-6) = +5$$

Select the letter next to the set that contains the correct answer.

(A) $\{+1 , +2 , +3 , +4 , +5\}$

(B) $\{-1 , -2 , -3 , -4 , -5\}$

(C) $\{+11 , +12 , +13 , +14 , +15\}$

(D) $\{-11 , -12 , -13 , -14 , -15\}$

Very good. You made the correct choice.

It is not necessary that we construct a NUMBER LINE. It is a basic concept to which we can refer for a check or a help.

Eventually, we expect you to use the technique of equation solving and to develop rules for handling signed numbers.

We have to find the value of the letter g, which will make

$$-4 + g = -16$$ a true statement

Recall the rule:

$$a + (-a) = 0$$

That is, a number plus its opposite is equal to zero.

Let us add the opposite of -4 to both members of the equation. By the addition property of equality, we get the equivalent equation,

$$-4 + (4) + g = -16 + (4)$$

$$0 + g = -16 + 4$$

$$g = -12$$

Please go on to question 15 below.

Question 15

Apply the proper principle and find the solution set of the equation

$$|x| + |-2| = 7$$

Select the letter next to the correct answer.

(A) $\{9\}$

(B) $\{5\}$

(C) $\{5, -5\}$

(D) $\{-9\}$

196
1

Very good. You made the correct choice.

The null set is the correct answer.

Let us see why. The expression $|y|$ is always a positive number.

That is

$$|y| > 0$$

$$|4| = 4$$

Adding any positive number to the positive number 4 , cannot give a sum of 3 . Thus, the solution set of this equation is the null set.

Please go on to question 17 below.

196
2

Question 17

Apply your knowledge and find the solution set of the following equation:

$$b - |18| = |-20|$$

Select the letter next to the correct answer.

(A) $\{18\}$

(B) $\{2\}$

(C) $\{38\}$

(D) $\{-2\}$

Good. Since the domain of x is the positive integers, we can just convert from absolute value to the actual value.

$$P = |-x| = x$$

$$Q = |-2x| = 2x$$

$$R = -|x| = -x$$

$$S = -2|x| = -2x$$

Whatever the value of x is, the four letters will have the same relative position on the NUMBER LINE.

Please go on to questior 19 below.

Question 19

Apply your knowledge and select the sets below that are closed under the operation of finding the absolute value.

$$S = \{4 , 3 , 2 , 1 , 0 , -1 , -2 , -3 , -4 , -5\}$$

$$T = \{-2 , -1 , 0 , 1 , 2 , \ldots\}$$

$$U = \{-3 , -1 , +1 , +3\}$$

(A) Set S

(B) Set S and T

(C) Set S and U

(D) Set T and U

198
1

Very good. You made the correct choice. Since the absolute value of a number is the same whether the number is positive or negative, the given equation

$$|x| + |-2| = 7$$

is equivalent to two equations.

Note: the absolute value of a number is the magnitude without the direction.

$|-2| = 2$

$|x| = x$

$|-x| = x$

Thus,

$x + |-2| = 7$ and $-x + |-2| = 7$

$x + 2 = 7$ $\swarrow -2$ $\qquad -x + 2 = 7$ $\swarrow -2$

$x = 5$ $\qquad -x = -5$ $\swarrow \cdot(-1)$

$x = -5$

Thus there are two answers: $x = 5$

$x = -5$

We can check:

let $x = 5$

$|x| + |-2| = 7$

$|5| + |-2| \stackrel{?}{=} 7$

$5 + 2 \stackrel{?}{=} 7$

$7 = 7$ ✓

let $x = -5$

$|x| + |-2| = 7$

$|-5| + |-2| \stackrel{?}{=} 7$

$5 + 2 \stackrel{?}{=} 7$

$7 = 7$ ✓

Please go on to question 16 below.

198
2

Question 16

Apply your knowledge and find the solution set of the following equation:

$$|y| + |4| = 3$$

Select the letter next to the correct answer.

(A) $\{-1\}$

(B) $\{0\}$

(C) $\{-7\}$

(D) The null set, $\emptyset$

Yes, your choice is correct.

By the CLOSURE PROPERTY under the operation of absolute value, we mean that for every element in a set, the absolute value of this element must also be in the set. Now.

$$T = \{-2 , -1 , 0 , 1 , 2 , ...\}$$

Consider each element of T .

$$|-2| = 2$$

$$|-1| = 1$$

$$|0| = 0$$

and although the set extends indefinately in the positive direction, and absolute value of all the rest of the positive integers are the same positive integers. Thus, T is closed under the operation of absolute value.

Similarly, set U is closed since

$$|-3| = 3$$

$$|-1| = 1$$

and

$$|0| = 0$$

Please go on to question 20 below.

Question 20

Relate to the proper principle and find the solution set of the following equation

$$|x - 1| = 3$$

Select the letter next to the correct answer.

(A) $\{4 , 1\}$ (C) $\{2 , 4\}$

(B) $\{2 , -2\}$ (D) $\{-2 , 4\}$

Very good. You made the correct choice.

We know that the absolute value of a number is the same as the absolute value of its opposite. Thus,

$$|x - 1| = |-(x - 1)|$$

Since these two expressions are the same, the given equation is equivalent to the following two equations,

$$x - 1 = 3$$

and

$$-(x - 1) = 3$$

We find the solution set of the first equation by using the addition property of equality. We have,

$$x - 1 = 3 \quad \text{+ } 1$$
$$x = 4$$

To find the solution set of the second equation,

$$-(x - 1) = 3$$

we note that if two numbers are equal, their opposites are equal. Hence,

$$-(x - 1) = 3 \quad \cdot(-1)$$
$$x - 1 = -3 \quad +1$$
$$x = -2$$

You have now finished this segment. Hand in your PUNCH CARD.

You should have entered in your NOTEBOOK the following definitions and formulas:

(1) For every directed number a, there exists a unique number $-a$, such that

$$a + (-a) = (-a) + a = 0$$

(2) The opposite (additive inverse) of a sum of two numbers is the sum of their opposites.

(3) The absolute value of a, written $|a|$ equals a if $a > 0$ and $-a$ if $a < 0$.

or $|+a| = a$

$|-a| = a$

You should now be able to complete the remaining problems of your HOMEWORK ASSIGNMENT in preparation for a test on Volume 4.

PROGRAMMED MATHEMATICS CONTINUUM
ALGEBRA - LEVEL ONE

ERRATA SHEET
VOLUME 4

12/19/69
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To the users of this book:

Computer analysis of the student's performance in his progress through this book will have as one of its purposes the collection of data indicating the need for revision of the material presented. Certain typographical errors already exist and will also be corrected. Listed below are misprints that will affect the mathematics of the problems. Make a careful correction of each misprint as follows:

PAGE	MISPRINT	CORRECTION	CHECK WHEN CORRECTION MADE
60	go on to question 4	go on to question 3	
68	Question 4	Question 3	
2	Ques. 11 $\div 2(K \div r)$	$2(K \div 4)$	